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PROBLEM SOLVING IN HIGH SCHOOL AND COLLEGE STUDENTS.

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THE THREE MAIN PURPOSES OF THIS RESEARCH IN PROBLEM-SOLVING WERE (1) TO EVALUATE GROUP PERFORMANCE VERSUS PERFORMANCE NORMS BASED ON THE PROPERTIES OF THE PROBLEM, (2) TO STUDY INDIVIDUAL PERFORMANCE, AND (3) TO INVESTIGATE THE EFFECT THAT A PARTICULAR EDUCATIONAL LEVEL HAS ON THE PERFORMANCE OF THESE PROBLEMS. THE SUBJECTS USED CONSISTED OF A GROUP OF 38 MALE EXPERIMENTAL SUBJECTS AND 38 MALE CONTROL SUBJECTS FROM HIGH SCHOOL AND COLLEGE. THE SUBJECTS WERE DIVIDED INTO TWO GROUPS, HIGH AND LOW SCORERS, AND SUBDIVIDED INTO HIGH AND LOW ACADEMIC GRADES. THE RELATIONSHIPS BETWEEN PROBLEM-SOLVING ABILITY, AND INTELLECTUAL AND EDUCATIONAL LEVEL WERE EXPLORED. THE CONCLUSIONS ON SCHEMATA NORMS WERE SHOWN TO BE MORE USEFUL THAN THE GROUP NORMS AS A PROCESS FOR PROBLEM SOLVING. THE EXPERIMENTAL GROUP PERFORMED BETTER THAN THE CONTROL GROUP ON INDIVIDUAL PERFORMANCE CURVES. THE DEVELOPMENT OF NEW PROBLEMS AND METHODS OF SCORING PROBLEM-SOLVING PERFORMANCES WAS SUCCESSFUL AND ALLOWED FOR MORE MEANINGFUL WAYS OF OBTAINING INFORMATION. FURTHER RESEARCH REQUIRES THE DEVELOPMENT AND PREPARATION OF BETTER INSTRUMENTS TO STUDY THE THINKING PROCESSES OF STUDENTS. (RS)

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PROBLEM SOLVING IN HIGH SCHOOL AND COLLEGE STUDENTS

Cooperative Research Project No. 2199

by

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Introduction and Acknowledgements

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This report includes three different studies listed under Phase I, Phase II and Appendix III.

Phase I is the result of a further analysis of data gathered under CRP No. 1449 and was published separately under the title of "Individual Differences in Thought Processes", Loyola Psychometric Laboratory, Pub. No. 33, 1963. This study was the subject matter of a Ph.D. dissertation by H. Fogliatto.

While Phase I is a continuation of previous research, Phase II is an attempt to verify previous findings using new problems, new subjects and new methods of analysis. Several persons were involved in this Phase of the study. Dr. Fogliatto contributed with the preparation of some new problems and partially to the analysis of the data. Mr. J. Erdmann contributed to the analysis and interpretation of the results. Mr. M. Donnelly has been responsible for coordinating the data and performing many of the statistical tests that were used. Mrs. P. Dore was responsible for the original tabulation of part of the data, and Mr. G. Burger for adapting the data to computer operation.

Appendix III is a methodological study to compare different techniques of scoring problems. This was the subject matter of a Master's thesis performed by Mr. J. Erdmann, partially supported by this grant. This study has already been published as Loyola Psychometric Laboratory, Publication No. 40, 1964.

Other members of the Loyola Psychometric Laboratory contributed in different ways to the preparation of this report. We are grateful for their cooperation. Special mention should be made of the final typing of the report, carefully performed by Miss Rita M. Schneider and Miss Patricia L. Folly.

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H. J. A. Rimoldi

II

Problems on Which the Research Was Focused

This study is a continuation of research on problem solving that has been conducted at the Loyola Psychometric Laboratory on grants CRP No. 1015 and CRP No. 1449.

Two major problems were involved: a) A thorough analysis of the data gathered in CRP No. 1449, in order to identify tactics which characterize performance in problem solving situations; this part of the study is given in Phase I of this report. b) The evaluation of the effects of educational level and ability in the performance of problems prepared to study problem solving processes. This corresponds to Phase II of this study.

For Phase II new problems were prepared according to specifications arising from our previous research. These refer to defining various properties of the individual problems related to their formal relationships (schemata) and to their content. This makes it possible to compare the performances of subjects belonging to different educational and ability levels by means of a standard independent of group performances. In order to characterize the groups of subjects as such a study of those groups was made and this was evaluated in terms of the standard.

On the basis of the logical structure of the problem, objective limits of performance can be defined and the observed group performance can be evaluated in terms of these limits.

As shown by Erdmann (1964) in Appendix III of this report the "pulling out" method for scoring these problems seems to differentiate between poor and good problem solvers much better than techniques based on group performances. In Phase II a further validation of the scoring technique was attempted.

The differential effects of academic grades and tests of problem solving ability as discriminators of performance in the experimental problems at various levels of education were investigated. This problem was analyzed using different approaches.

Finally, new ways of characterizing individual performances suggested themselves in the course of this research. These are persued in Phase II.

Some of the earlier approaches for the evaluation of problem solving performances developed by us have been utilized to obtain corroborative evidence as to their usefulness and validity. These are not necessarily detailed in the following text, since previous research has indicated them to be of questionable value. It is expected that the methods here used will prove to be satisfactory in further studies.

III

Objectives and/or Hypotheses

The main objectives of this study are the following:

1) A thorough analysis of the data gathered in CRP 1449.

2) Critical testing of previous findings using an independent group of subjects at different educationl and ability levels.

3) Development of new approaches to study the processes followed by subjects in solving the experimental problems. This was pursued in order to obtain both new group and individual measures.

Problems related to these major objectives are:

a) Analysis of problems with varying complexity in the logical structure and different contents, in terms of various evaluations including length of plateaux.

b) Comparison of the performance of subjects at the high school and college levels. These groups were further characterized in terms of their academic grades and performance in standard tests of intelligence and problem solving ability.

c) Identification of individual tactics and their mutual relationships, as seen in convex sets.

d) Validation of the pulling out method of scoring in terms of independent criteria.

e) Characterization of group performances and individual performances in terms of uncertainty.

f) Comparison of these evaluations with objective limits of performance based on the logical analysis of the problems.

g) Comparisons of problems with varying content but similar logical structure at different levels of academic achievement in two educational levels.

IV

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CHAPTER I

INTRODUCTION

Most of the studies to appraise individual differences using mental tests have been concerned with the development of group norms. Conclusions about an individual were attempted by reference to these norms or other statistics which by definition are based on averages for the group. The analysis of test results often pivots on the properties of responses to items that are classified as correct or incorrect.

In several studies (Rimoldi, Devane, 1961; Rimoldi, Meyer, Meyer, Fogliatto, 1962; Rimoldi, Haley, Fogliatto, 1962; Fogliatto, 1962; Rimoldi, Fogliatto, Haley, Reyes, Erdmann, Zacharias, 1962) it was found: 1) that the process employed in solving a problem cannot be characterized only by the final answer, 2) that by using group norms we may be unnecessarily eliminating important individual differences. These studies presented ways of preparing problems to appraise thinking ability.

In a recently published study (Rimoldi, Haley, Fogliatto, Erdmann, 1963) it was reported that it is important for the experimenter to be able to control the schemata of the problems as well as their content (see procedure). This made possible the development of new ways of scoring these problems. Being able to control the schemata and the content, it is possible to score individuals in relation to these. A comparison, then, can be made between the performance of an individual score in terms of schemata and content as well as in terms of the norms established by the group. This has been one of the problems investigated in this dissertation.*

In previous research (Rimoldi, Devane, 1961; Rimoldi, Fogliatto, Haley, Reyes, Erdmann, Zacharias, 1962) it was found that training in problem solving improves the thinking process. In solving the problems, experimental subjects used fewer questions than the controls. More agreement was observed among the experimental subjects than among the control subjects as to the questions selected in order to solve the problems. The second problem undertaken in this research has been an investigation of the differences in the problem solving process between each experimental subject and the corresponding control subject who were matched according to specific criteria before the experiment.

In the study "Training in Problem Solving" (Rimoldi, Fogliatto, Haley, Reyes, Erdmann, Zacharias, 1962) it was reported: 1) that college students as a group select fewer questions in order to solve a problem than the high school students, 2) that the college group improves more under training than the high school group. A third problem investigated in this research was the importance of the educational level as a factor in problem solving performance.

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* This research is a systematic exploration of issues raised by previous research (Rimoldi, Fogliatto, Haley, Reyes, Erdmann, Zacharias, 1962).

Summarizing, the three main purposes of this research are: 1) To evaluate group performance versus performance norms based on the properties of the problem as well as the interaction of schemata and content. 2) To study individual performance by comparing the process of subjects with training to subjects without training individually matched before the experiment. 3) To investigate the effect that a particular educational level has on the performance of these problems.

CHAPTER II

REVIEW OF RELATED RESEARCH

A. Definition and classification of Problem Solving.

Since the early contribution of Wertheimer (1945) a number of studies have been published in this area.

Wertheimer considers the distinction between productive and reproductive thinking as most important when viewing work of an academic nature. In a problem situation there is a goal, obstacles to reaching the goal, and no clear perception of the means of obtaining it. Wertheimer presented his problems to children and adults. His contribution is the first study of the thought process that develops its conclusions from concrete examples. His approach differs from the present study. He presents a qualitative evaluation of the thought process. The present study is an attempt to evaluate quantitatively individual differences in thought processes.

According to Duncan (1959) thinking is most frequently defined as the integration and organization of past experience, while problem solving is defined as the discovery of correct response. Problem solving is considered to be fairly high on the discovery dimension, and this will be the distinction of problem solving from conditioning and rote learning, which are presumed to involve relatively little response discovery.

Underwood (1952) presents three methods for determining the amount of overlap between conditioning and thinking.

Bloom and Broder (1950) describe the difficulties of attempting to discover the nature of mental processes through retrospection, introspection, or the construction of test situations in which each of a variety of methods of attack would be reflected by a different solution. They classified the students as successful or unsuccessful according to their aptitude scores and marks on comprehensive examinations. The successful problem solver showed greater ability to understand the nature of the problem and to attack it in its own terms. The unsuccessful problem solvers showed lack of comprehension of direction and often presented solution of a problem other than the one that was expected.

Bruner, Goodnow, and Austin (1956) described classes of equivalence categories. "Functional categories" include at least those problem solving tasks where the subjects must categorize an object as fitting a certain function. They also suggest that defining attributes are sometimes combined to create either new or empty categories, and that those types of combination often occur in problem solving. They attempt to relate two major areas of thinking research, i.e., problem solving and concept formation.

Tate, Stanier and Harootunian (1959) classified students as good and poor problem solvers using as criteria their performance in a battery of tests, one of which was the "Thought Problems", a test that was especially prepared for that study. They concluded that the "good problem solvers are significantly better than the poor in nearly all tests where quality of response, accuracy, or judgment is required; and that, without exception, the more complex the task or the more restricted the requirements, the greater their superiority."

In the present study no attempt has been made to categorize the problem solvers. The Thought Problems have been used here for matching purposes. Individual differences has been evaluated using a different approach.

B. Training in problem solving.

A considerable number of studies have discussed ways of training people in problem solving tasks. Adams (1954) has found that a group of subjects trained on repeated presentations of the same problem were more efficient in solving new problems of the same class than a group trained on a number of different problems. Harlow (1949) held that training on a number of different problems will develop new ideas in the way of how problems should be attacked. This means that such a training will help the subject in the new situation.

Schroder and Rotter (1952) used a card sorting task with four groups of subjects and they altered the training in "the expectancy of change" given from group to group. According to the authors it is the training in "expectancy of change" which is required, and no training in a single solution that will solve the problem in the present situation.

Duncker (1945) conducted a study with educated adults. They were presented with arithmetic and geometric problems. In solving the problems the subjects had to analyze what was given and what was required. The process of solving a problem consisted in the generation and testing of hypotheses. Past experience plays an important role in the solving of new problems. The inability to use an object for a strange purpose in a given situation may be due to the previous use made of that object. Previous experience can have a negative effect when new problem situations are faced. Birch and Rabinowitz (1951) have also showed this effect and Adamson (1952) repeated three of Duncker's experiments with the same results.

Parnes and Meadow (1960) compared experimental subjects with control subjects matched for vocabulary ability on six creative ability tests. They reported differences statistically significant. The increase of productivity in the creative thinking process produced by the creative problem-solving course persisted for a period of at least eight months after the completion of the course.

Sommer (1960) reported a study with two groups of subjects. The experimental group received, before the experiment, correct solutions to problems involving the same principle as those in the experimental situation. The results showed: 1)wide differences between the two groups on the process leading to the solution; 2) once the solution to a problem has been experienced, it exerts a profound influence on the approach to similar problems; and 3) the use of a learned principle becomes more difficult if the problem is presented in a confusing manner. Blumenfeld (1956) reported two studies using geometrical theorms in which he changed the orientation and the figure. Buswell (1956) attempted to define common patterns in the solution of problems. The subjects were asked to discover a rule for arriving at the solution of an addition problem without the use of simple addition. The subjects found the problem very difficult and the results indicated more diversity than similarity in the problem solving approach. When the effect of training was tested in similar problems, about half of the subjects showed transfer.

In a study with high school students Rimoldi and Devane (1961) found that the experimental subjects - the group of subjects who went through a training period in problem solving - had a greater gain in mathematics grades than the control subjects.

A recent publication by Rimoldi, Fogliatto, Haley, Reyes, Erdmann, and Zacharias (1962) reported a research conducted with high school and college students. The design of the experiment permitted the testing of the influence that training in one type of problems would have on another type of problems. Transfer of training was found. It was also found that the subjects with training in problem solving use fewer questions and show more agreement among themselves as to which question they should ask in order to solve the problem than the control subjects. Similar results were also found when the trained subjects were introduced to new problems.

The experiments described in the previous paragraphs have dealt with the effect of prior experience. It has been demonstrated that training in a particular type of problems leads to maximum efficiency as long as the problem requires a similar solution. When different kinds of problems have to be faced, a wider training with emphasis on the need for change seems to be advisable. In the present research the effect of training has been studied at the individual level both in problems that require a similar solution and in completely different problems.

C. Presentation of the problems.

A number of studies have been reported in which the same problem has been presented under different modes or appearances. Many problems have been presented in either symbolic or concrete form with various degrees in between. Several studies have found no effect of varying concreteness on the problem. Saugstad (1957) in a repetition of Maier's experiment found that the miniature scale model did not call more attention to ceiling than the real presentation of the two pendulum problems. The same was reported by Lorge, Tuckman, Aikman, Spiegel, and Moss (1955a, 1955b) when they used the mined road problem at seven levels of reality (verbal, photographic, miniature scale model or real presentation, or various amounts of manipulation of the scale and real versions).

On the other hand contrary results have been reported. Cobb and Brenneise (1952) reported that "anchor real and extension solution of the two-string problem" decreased as concreteness [illegible]creases over four steps. Gibb (1956) tested children in subtraction problems with three degrees of concreteness. He found significant differences and no interaction. Rimoldi, Fogliatto, Haley, Reyes, Erdmann, Zacharias (1962) have reported a significant interaction (schemata-content) using problems with three different schemata and four contents for each one of them. In the present project, problems with three degrees of complexity and four degrees of concreteness has been studied.

D. The importance of age in problem solving.

Several studies have reported that age is an important variable in most types of problem solving. Sato (1953) working with children and adults had found that the former were more affected by the amount of training than by the difficulty level of the problems, while the reverse was true for adults. Hunter (1957) reported that 16 year olds did better than 11 year olds on his syllogistic-like problems. Moraes (1954) found different patterns of thinking among school children of different ages on arithmetic reasoning problems. Rimoldi, Fogliatto, Haley, Reyes, Erdmann, Zacharias (1962) found that college freshmen use fewer questions in order to solve a problem than do high school freshmen. It was also found that the college students as a group improve more under training than high school students. In the present study the importance of educational level has been investigated using high school freshmen and college freshmen as subjects.

Rimoldi, Meyer, Meyer, Fogliatto (1962) report a research with graduate students (from 23 to 40 years old) in which the description and analysis of the sequential organization of complex process was studied (i. e., problem solving) and also how these change from early life to old age. New research, not yet reported, has made use of information theory in which a series of problems have been administered to subjects of varied age level (from 11 to 80 years old). Tentative results indicate that uncertainty in problem solving decreases with chronological age to the young adult level and then gradually increases.

E. Methodology.

Johnson (1955) discussed three techniques for the analysis of individual differences in thought processes. 1) comparison of groups (in respect to age, sex, education, or ways of attacking the problem). 2) correlational analysis (the time of solution, number of responses, number of right answers). 3) factor analysis.

A technique similar to the one to be used in this study was devised by Bryan (1954) for evaluating electronic trouble shooting. Glaser, Damrin, and Gardner, (1954) presented a similar technique, the Tab Item Technique, which was also used in electronic trouble shooting. John and Rimoldi (1955) and John (1957) studied the sequential properties of complex reasoning by means of the Problem Solving and Information Apparatus. This apparatus may be useful in studying certain phases of abstract reasoning, but cannot be used in a variety of situations where less abstract problems are examined.

The technique used in this study was devised by Rimoldi (1955). The technique was first applied to study mental processes in medical students. A series of studies related to this approach have been published by the Loyola Psychometric Laboratory over a period of several years (Rimoldi, 1960, Rimoldi, 1961, Rimoldi, Devane, and Haley, 1961). A final report by Rimoldi, Haley, Fogliatto (1962) summarized the whole work. This approach has been applied to other areas than medicine (Tabor, 1959, Mohrbacher, 1960, Gunn, 1961, Rimoldi, Meyer, Meyer, Fogliatto, 1962, Fogliatto, 1962). The same technique has been used to evaluate the effect of training in high school students (Rimoldi and Devane, 1961) and in high school and college students (Rimoldi, Fogliatto, Haley, Reyes, Erdmann, and Zacharias, 1962).

The studies described in the previous paragraphs deal with evaluation of the subjects' performances using group norms. In the present research the performance of the subjects in problem solving has also been studied using schemata norms as described by Rimoldi, Haley, Fogliatto, Erdmann, 1963).

CHAPTER III

PROCEDURE

A. Design of the experiment:

1. Pre-testing sessions: every subject whether control or experimental received at the beginning of the experiment 3 problems of type c. (see problems).

2. Training sessions: the experimental subjects (high school and college) completed at least 24 problems - 12 of type a and 12 of type b. (see problems).

3. Post-testing sessions: every subject whether control or experimental received at the end of the experiment:

   a) the same 3 problems of type c that were administered at the beginning.

   b) 2 problems of type a similar to the ones used in training sessions.

   c) 2 problems of type b similar to the ones used in the training sessions.

   d) 2 new problems completely different from the ones used in the training sessions (1 of type a and 1 of type c).

B. Subjects:

The subjects used in this study consist of a group of 38 experimental subjects (19 male high school and 19 male college freshmen) and 38 control subjects (19 male high school and 19 male college freshmen).

1. The high school subjects were selected among the freshmen of St. Ignatius High School, Chicago, Illinois, if they had an I.Q. of 118 or above on the Henmon-Nelson Test of Mental Abilities. On this basis seventy students were selected. The Raven's Progressive Matrices Tests and Thought Problems, Part 1, were administered to all of them. Nineteen experimental-control pairs were selected and each pair matched for I.Q. and for the score on the Raven Progressive Matrices Test. The subjects after being matched were randomly assigned to be a control or experimental subject.

2. For the college subjects, 50 were selected among the freshmen of Loyola University College of Arts and Sciences. The Raven Progressive Tests and Thought Problems, Part I, were administered to all of them. Using their scores on these two tests, 19 experimental-control pairs were selected. Each member of the pair was randomly assigned to be an experimental or control subject. For the college students, it was not possible to match them according to I.Q. because school records could not easily be compared.

The Mean, Standard Deviations and Number of Subjects for the I.Q.'s, the Raven's Progressive Matrices Tests, and the Thought Problems, Part I, are presented in Table I.

Table 2 presents the correlation for the matched pairs.

TABLE I

MEAN, STANDARD DEVIATION AND NUMBER OF SUBJECTS FOR RAVEN, I. Q. AND THOUGHT PROBLEMS, PART I, FOR HIGH SCHOOL AND COLLEGE STUDENTS.

	HIGH SCHOOL						COLLEGE					
TESTS	CONTROL			EXPERIMENTAL			CONTROL			EXPERIMENTAL		
	M	σ	N	M	σ	N	M	σ	N	M	σ	N
RAVEN	49.74	4.26	19	48.84	4.25	19	51.84	5.01	19	52.53	4.49	19
I.Q.	125.21	7.30	19	125.68	7.38	19						
THOUGHT PROBLEM I.	11.29	3.39	19	11.91	3.99	19	13.24	3.56	19	13.67	3.40	19

TABLE 2

MATCHED PAIR CORRELATIONS FOR HIGH SCHOOL AND COLLEGE STUDENTS

HIGH SCHOOL	RAVEN TEST........................	.94
	HENMON-NELSON I.Q................	.98
COLLEGE	RAVEN TEST........................	.89
	THOUGHT PROBLEM, PART I...........	.82

C. Problems:

Three different types of problems are used in this research. Every problem was individually administered to the subjects in all the sessions.*

1. Problems of type a.

The subject is presented with a problem and a set of questions from which he may select as many questions as he wishes and in any order that he desires. Each question is presented on a separate card. The answers are presented on the reverse side of the cards. When the subject thinks he has enough information, he stops selecting questions and gives his answer. He records the questions that he has asked in the corresponding order as well as the answer.

Problems of type a are problems 31, 33, and 35. The numbers refer to degrees of complexity in the schemata.

For problems 31 the schemata can be represented as a tree

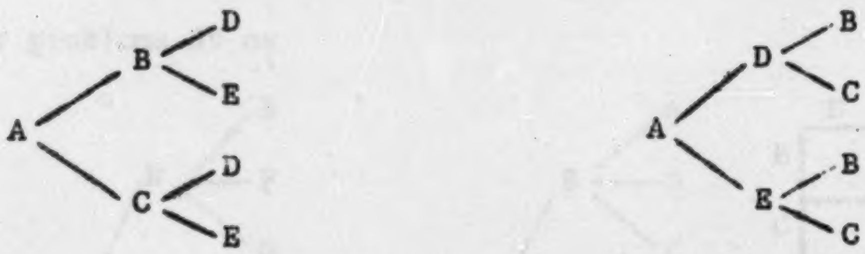

FIGURE I

or a fourfold table with degrees of freedom

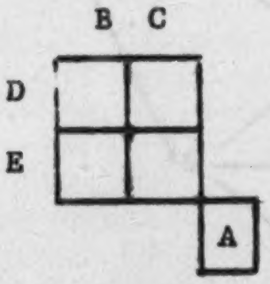

FIGURE 2

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* For a complete description of the problems found in this research refer to publication "Training in Problem Solving" (Rimoldi, Fogliatto, Haley, Reyes, Erdmann and Zacharia, 1962)

For problem 33 as

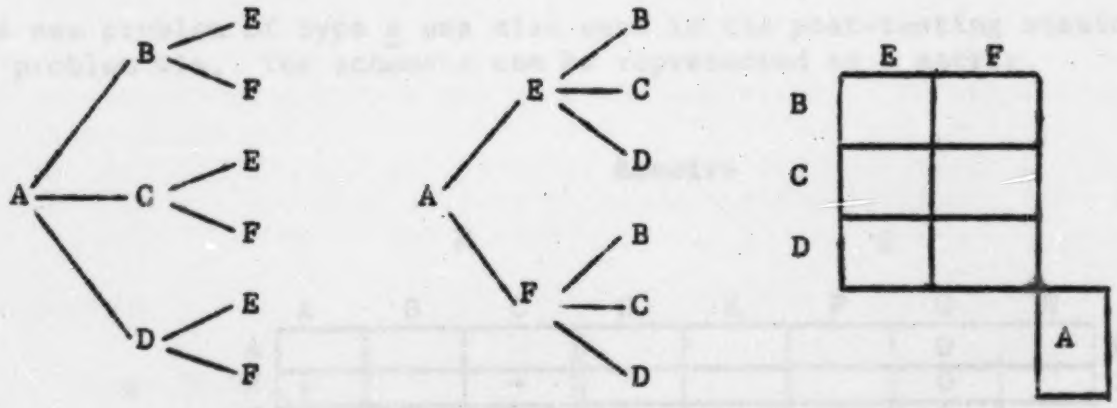
A
B
C
D
E
F
E
F
E
F
A
E
F
B
C
D
B
C
D
E
F
B
C
D
A

FIGURE 3

For problems 35 as

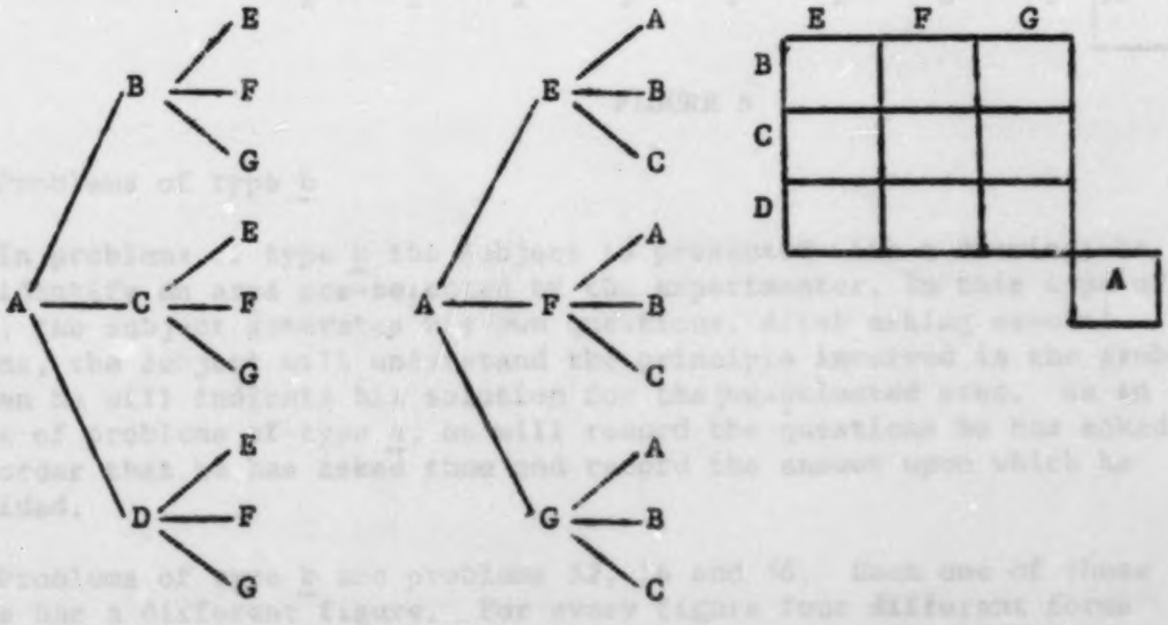
A
B
C
D
E
F
G
E
F
G
E
F
G
A
E
F
G
A
B
C
A
B
C
A
B
C
E
F
G
B
C
D
A

FIGURE 4

Each of these problems has four forms. In form A the problem is presented in a concrete way. Form B is an abstract presentation of the problem. Form C is a negative presentation of the problem. In form D the answers are given in letters instead of using numbers as in forms A, B, and C.

At least 12 problems of type a were completed by the experimental subjects during the training sessions. In the post-testing sessions the 2 problems of type a were: 31D' and 35B'. They have the same schemata as 31 and 35 and the content of form D and B respectively.

A new problem of type a was also used in the post-testing sessions; this is problem 41A. The schemata can be represented as a matrix.

Receive

			N				S			
		A	B	C	D	E	F	G	H	
	A							0		0
N	B	+		+				0		2
	C				+	+		0		2
Initiate	D							0	+	1
	E	+	+	+	+		+	0		5
S	F							0		0
	G	0	0	0	0	0	0	0	0	0
	H							0		0
		2	1	2	2	1	1	0	1	10

FIGURE 5

2. Problems of type b

In problems of type b the subject is presented with a drawing; he has to identify an area pre-selected by the experimenter. In this type of problem, the subject generates his own questions. After asking several questions, the subject will understand the principle involved in the problem, then he will indicate his solution for the pre-selected area. As in the case of problems of type a, he will record the questions he has asked in the order that he has asked them and record the answer upon which he has decided.

Problems of type b are problems 32, 34 and 36. Each one of these problems has a different figure. For every figure four different forms (A, B, C, and D) were developed. Form A is the most simple and form D the most complex with two degrees of complexity in between B, and C. Every subject in the experimental group completed at least 12 problems of type b. In the post-testing sessions, 2 problems of type b were administered (32F and 36F). They have the same figure as problems 32 and 36 respectively. Nevertheless, for problem 32F the principle involved was varied. When problem 32 was used in the training sessions, the underlying rationale was a series of letters or numbers of combinations of both following a horizontal pattern. In problem 32F a series of numbers was used but followed a vertical pattern. For problem 36F the same figure and content as the one used in the training sessions was kept.

3. Problems of type c.*

In problems of type c the subject is presented with a problem and a set of questions, or a figure and a set of questions. The subject proceeds in the same way as in problem of type a. Problems of type c are problems 1, 19, and 25 which were used in the pre and post-testing sessions and problem 26 which was used only in the post-testing sessions. Problems 1 and 25 are figure problems and problems 19 and 26 are word problems.

D. Methodology.

1. Scoring methods:

a) Group Norms:

The subjects were scored in terms of group norms using a technique devised by Rimoldi (1960). This technique utilizes the frequency of selection of a specific question in a particular order. These frequencies are converted to proportions to indicate the percentage of the total group that respond using a specific question in that order. In order to score a subject the proportion corresponding to every question asked is accumulated in the corresponding order. This gives the observed score (O).

Proportions for every card in every possible order are also computed on the basis of randomness. By subtracting these proportions (E) based on randomness from the observed (O) proportions a table of (O-E) is computed. Using these proportions, it is possible to obtain a (O-E) score for every subject by accumulating the (O-E) proportions corresponding to any questions he asked in the corresponding order.

With these norms every experimental subject was scored in all the problems of the training sessions.

b) Schemata Norms:

Problems of type a:

A scoring method described by Rimoldi, Haley, Fogliatto and Erdmann (1963) was used for problems of type a in order to score the subjects in terms of schemata norms. This technique is based on the properties of the problems. This means that the frequency of selection of each question in a particular order is established in terms of the sequence of logical relationships involved. As in the previous method these frequencies are converted to proportions to indicate the percentage of the total possible selections (as indicated by the schemata) for that specific questions in that particular order.

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* These problems were available at the Loyola Psychometric Laboratory from previous research (Cooperative Research Project No. 1015)

This gives a table of observed proportions (O_s). A table (O_s-E) proportions is also computed. The procedure for scoring the subjects is similar to the one used with group norms.

The experimental subjects were scored using these norms on the 12 problems of the training sessions and on the 3 problems of the post-testing sessions. The control subjects were scored on the 3 problems of the post-testing sessions.

Problems of type b:

For this type of problem there is no pre-established sequence of questions to be asked in order to solve the problem. The subjects originate their own questions. They can start asking about the areas in any order they want or according to some possible sequence that they may discover by inspection of the figure. For example, some figures have lines of different colors or of different kinds (straight, curved, dotted, etc.) or a combination of both. This could suggest to the subject that there is some relationship between the color and kind of lines and the answer to the problem. Nevertheless, this does not always happen. Problem 32 (A, B, C, D) have different kinds of lines and the rationale involved has nothing to do with it. In problem 34 (A, B, C, D) different color and kind of lines were used and the answer depends only upon the color. Problem 36 (A, B, C, D) has the same figure with different colors and kinds of lines. In form A only the colors are important, for form B the answer depends only on the kinds of lines and in forms C and D the answers depend upon the colors and kinds of lines. This means that the subject should try different approaches before finding the solution. It is not possible to say that one approach is better than the others. But, it is possible to limit the number of questions that are necessary in order to solve the problem. On this basis the so called "schemata norms" have been developed for every problem of type b. Using this approach the subjects will receive a score on each question asked until he completes the necessary number of questions needed to solve a problem. After that for every question he asks, he will receive a score of zero. This means that when the performance curves are plotted, the plateau will be always found at the end of the curve. Using a similar approach as on problems of type a (O_s) and (O_s-E) score is given to each subject.

The score obtained by accumulating the proportions for all the questions necessary in order to solve the problem (provided that the subject has asked at least the minimum number) is divided by the total number of questions asked. If the subject asks less than the required number of questions, he will receive a score for every question he has asked; but, in order to find his final score the cumulative sum of scores will be divided by the specified number of questions.

Using this approach every experimental subject has been scored on every one of the 12 problems used in the training sessions and on the 2 problems of the post-testing sessions. The control subjects have been scored on the 2 problems of the post-testing sessions.

Problems of type c:

The schemata norms for the problems of type c are based on the principle of the process of elimination. This means that the question that eliminates the largest number of areas (in the case of a figure) or possible answers (where it applies in the case of a word problem) should be asked first. The question eliminating the next largest number of areas or answers should be chosen second, and so on until the final solution is reached. After the sequence has been developed, it is processed the same as problems of type a.

Every subject whether control or experimental has been scored using these norms on the 3 problems of the pre-testing sessions and on the 4 problems of the post-testing sessions.

2. Performance curves:

The performance curves indicate the subject's approach to the problem. They are obtained by plotting the score of the subjects at each step. The way that a subject has attacked the problem can be compared with the tactic used by another subject. It is possible to compare the tactic of a control subject with the tactic used by his corresponding matched experimental subject. It is also possible to see the tactics that an experimental subject has followed throughout the tests in the training sessions.

Performance curves have been drawn for every control and experimental pair using schemata norms in the problems of the pre and post-testing sessions. (The performance curves of a control-experimental pair are presented in figures 26 to 33 inclusive).

For the experimental subjects performance curves have been drawn using both schemata and group norms for every problem of the training sessions.

Figures 14 to 25 present the performance curves for an experimental subject throughout all the problems of the training sessions in the order that he has received them.

When group norms are used for drawing the performance curves they will always show an increment on the curve because the scores are accumulated. Nevertheless, the degree of increment will depend on the group, that is, a question that has been selected by a larger number of subjects in the group will have a very high score and consequently the performance curve will show a large increment at that point. By the same token a card that has been selected by just one subject in the group will have a very small value even if the question is a very useful one.

When the performance curves are based on schemata norms, the curve will not always show an increment. There will be moments when the subject had selected a useless question and no increase will be shown on the curve. These plateaux can appear at any moment on the curve for problems of type a and c (See figures 26, 28, 29 and 33). For problems of type b, by the way that schemata norms have been defined, these plateaux will always appear at the end of the curve (See figures 31 and 32).

3. Convex sets:

The convex sets are obtained by plotting for every subject the (O_s) score on the abscissa and the (O_s-E) score on the ordinate (Rimoldi, Haley, 1962). Drawing successive lines from one extreme point to another, a poligonal convex set is defined in such a way that any corner of the poligonal convex set will represent a sequence followed by a subject; the other sequences or tactics will fall inside the convex set or on the boundaries. It is clear; that the convex set corresponding to the tactics followed by the experimental subjects will not necessarily coincide with the convex set that emerge from the tactics followed by the control subjects.

Convex sets have been drawn for every problem of the pre and post-testing sessions using schemata norms. It is possible to compare the performance of the control and the experimental subjects, and also to compare the college with the high school students. The convex sets for the high school and college students are presented in figures 34 to 57 inclusive. The numbers correspond to a subject who represents a tactic. The tactic on the boundaries of the convex sets are given at the bottom of the figures. Notice that the same score can be obtained using different tactics. But, the reverse does not hold; a tactic will always have the same score regardless of the subject who worked the problems.

CHAPTER IV

ANALYSIS OF THE DATA AND FINDINGS

A. Training sessions

1. Experimental subjects
   a) Group norms.

The problems used in the training sessions are 31A,B,C,D; 33A, B,C,D; 35A,B,C,D; 32A, B,C,D; 34A,B,C,D; and 36A,B,C,D.

After scoring every experimental subject on all these problems using group norms analyses of variances were performed separately for high school and college students as well as for problems of type *a* and problems of type *b*. Tables 3 and 4 present the results of the analyses of variances for the high school students in problems of type *a* and in problems of type *b* respectively. Similar analyses of variances were performed for the college students and the results are presented on tables 5 and 6.

Of great interest here is to test the effect that the complexity of the problem represented by the schemata and the effect the familiarity of the content have on the process of solving these problems. The interaction between schemata and content is also of interest.

The "F" ratio for the main effect schemata and for the interaction between schemata and content are significant at the .001 level in all cases. This means that the complexity of the schemata is a significant source of variation. The "F" ratio for the main effect content is significant at the .001 level for the college students in problems of type *a* and in problems of type *b*. For the high school students the "F" ratio is significant at the .01 level on problems of type *b* and not significant on problems of type *a*.

Figures 6 and 7 present the mean of the accumulative score (group norms) for high school students on problems of type *a* and on problems of type *b*. Similarly, figures 8 and 9 present the mean of the accumulative score for the college students. Inspection of all these figures show that the interaction between schemata and content is highly significant in every case.

TABLE 3

ANALYSIS OF VARIANCE FOR PROBLEMS OF TYPE a (TRAINING SESSIONS) ON THE HIGH SCHOOL STUDENTS BASED ON GROUP NORMS.

Source	Sum of Squares	df	Variance Estimate	F
Main Effects:				
Schemata	.023285	2	.011642	8.94^{xxx}
Content	.003281	3	.001093	1.06
Subjects	.017033	18	.000946	
Interaction:				
Schemata X Content	.021671	6	.003611	4.24^{xxx}
Schemata X Subjects	.046894	36	.001302	
Content X Subjects	.055450	54	.001026	
Interaction:				
Schemata X Content X Subjects	.092113	108	.000852	
Total	.259727	227		

xxx
$p < .001$

TABLE 4

ANALYSIS OF VARIANCE FOR PROBLEMS OF TYPE *b* (TRAINING SESSIONS) ON THE HIGH SCHOOL STUDENTS BASED ON GROUP NORMS

Source	Sum of Squares	df	Variance Estimate	F
Main Effects:				
Schemata	.063380	2	.031690	23.81[xxx]
Content	.008777	3	.002925	5.52[xx]
Subjects	.151939	18	.008441	
Interaction:				
Schemata X Content	.050606	6	.008434	10.06[xxx]
Schemata X Subjects	.047924	36	.001331	
Content X Subjects	.028644	54	.000530	
Interaction:				
Schemata X Content X Subjects	.090540	108	.000838	
Total	.441810	227		

[xx] $p < .01$

[xxx] $p < .001$

TABLE 5

ANALYSIS OF VARIANCE FOR PROBLEMS OF TYPE *a* (TRAINING SESSIONS) ON THE COLLEGE STUDENTS BASED ON GROUP NORMS

Source	Sum of Squares	df	Variance Estimate	F
Main Effects:				
Schemata	.147868	2	.073934	48.74^{xxx}
Content	.064169	3	.021389	29.50^{xxx}
Subjects	.035722	18	.001984	
Interaction:				
Schemata X Content	.125846	6	.020974	23.18^{xxx}
Schemata X Subjects	.054636	36	.001517	
Content X Subjects	.039191	54	.000725	
Interaction:				
Schemata X Content X Subjects	.097786	108	.000905	
Total	.565218	227		

^{xxx} $p < .001$

TABLE 6

ANALYSIS OF VARIANCE FOR PROBLEMS OF TYPE b (TRAINING SESSIONS) ON THE COLLEGE STUDENTS BASED ON GROUP NORMS

Source	Sum of Squares	df	Variance Estimate	F
Main Effects:				
Schemata	.029046	2	.014523	24.29^{xxx}
Content	.021903	3	.007301	11.52^{xxx}
Subjects	.039128	18	.002173	
Interaction:				
Schemata X Content	.057309	6	.009551	22.63^{xxx}
Schemata X Subjects	.021568	36	.000599	
Content X Subjects	.034289	54	.000634	
Interaction:				
Schemata X Content X Subjects	.045614	108	.000422	
Total	.248857	227		

^{xxx} $p < .001$

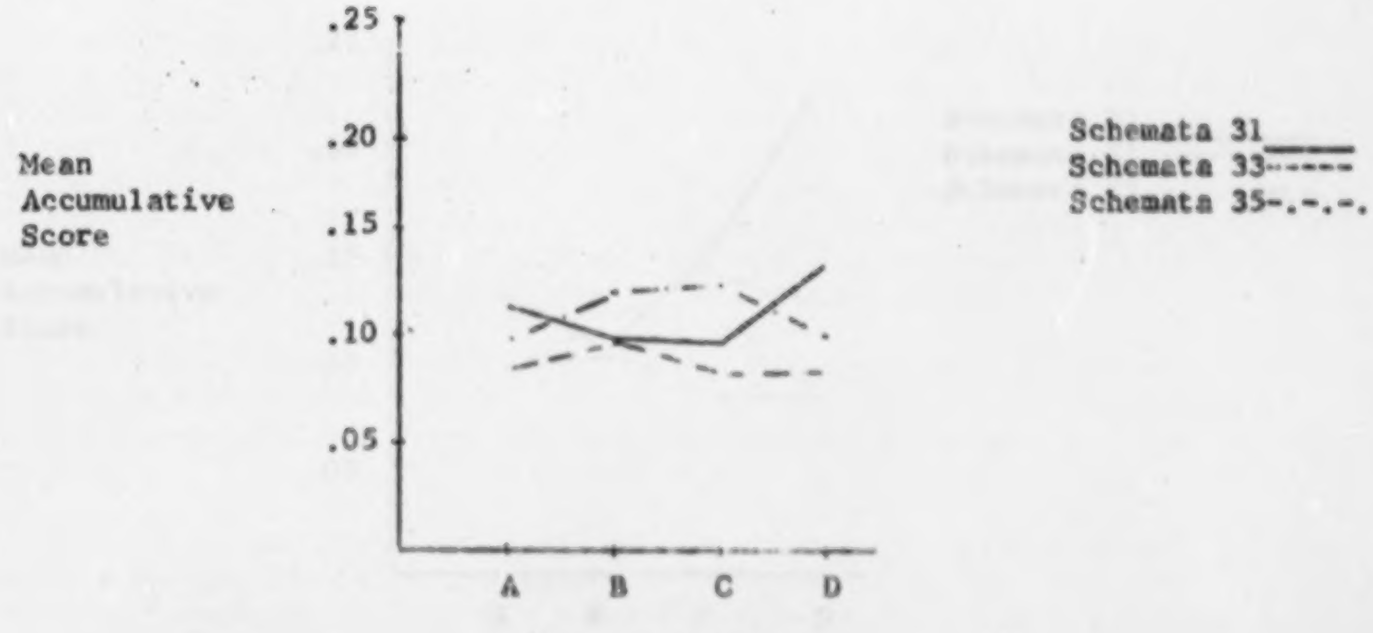

FIGURE 6

MEAN ACCUMULATIVE SCORE (GROUP NORMS) OF HIGH SCHOOL SUBJECTS ON PROBLEMS 31, 33, and 35 (TYPE a) FOR THE FOUR FORMS OF EACH PROBLEM.

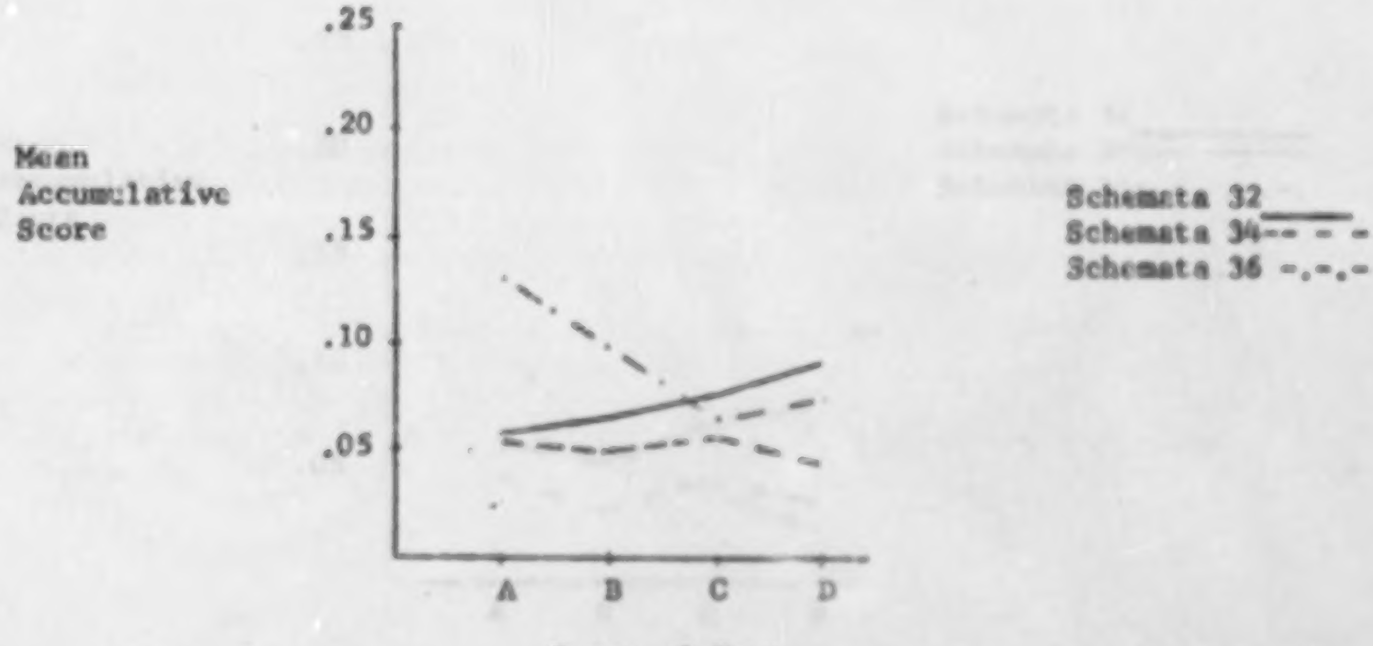

FIGURE 7

MEAN ACCUMULATIVE SCORE (GROUP NORMS) OF HIGH SCHOOL SUBJECTS ON PROBLEMS 32, 34, and 36 (TYPE b) FOR THE FOUR FORMS OF EACH PROBLEM.

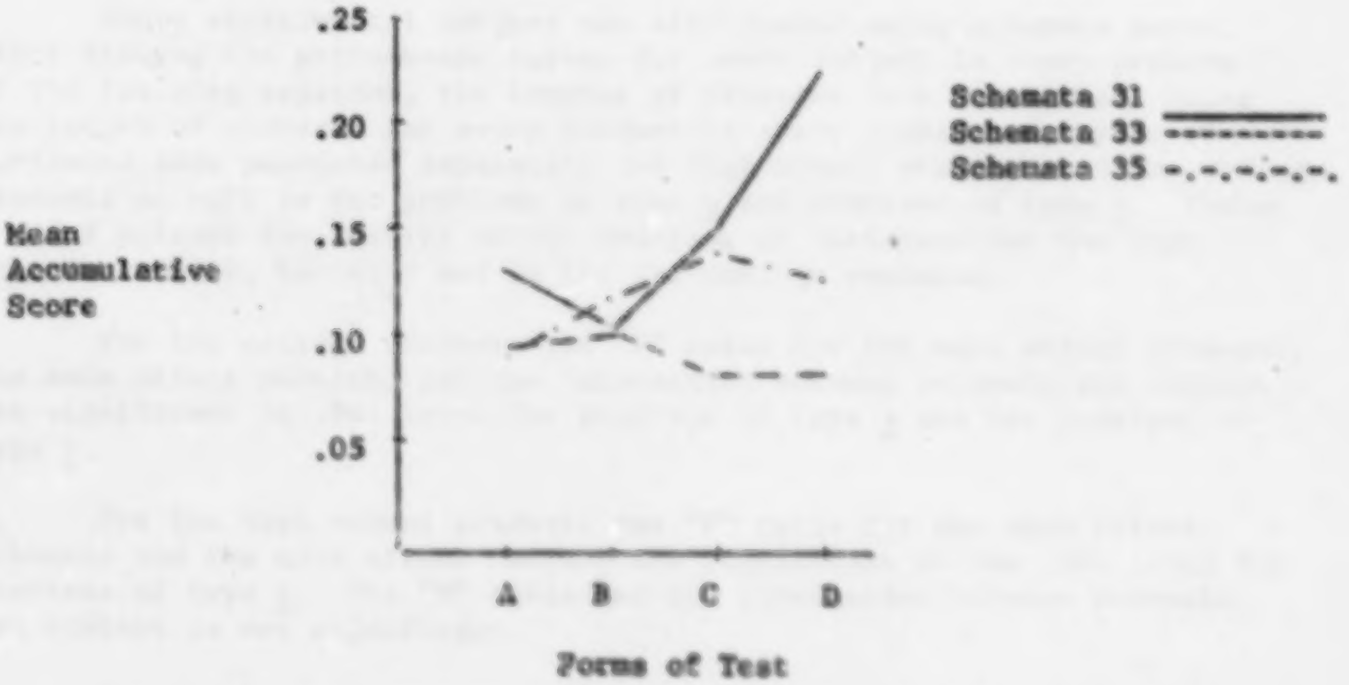

FIGURE 8

MEAN ACCUMULATIVE SCORE (GROUP NORMS) OF COLLEGE SUBJECTS ON PROBLEMS 31, 33, and 35 (TYPE a) FOR THE FOUR FORMS OF EACH PROBLEM.

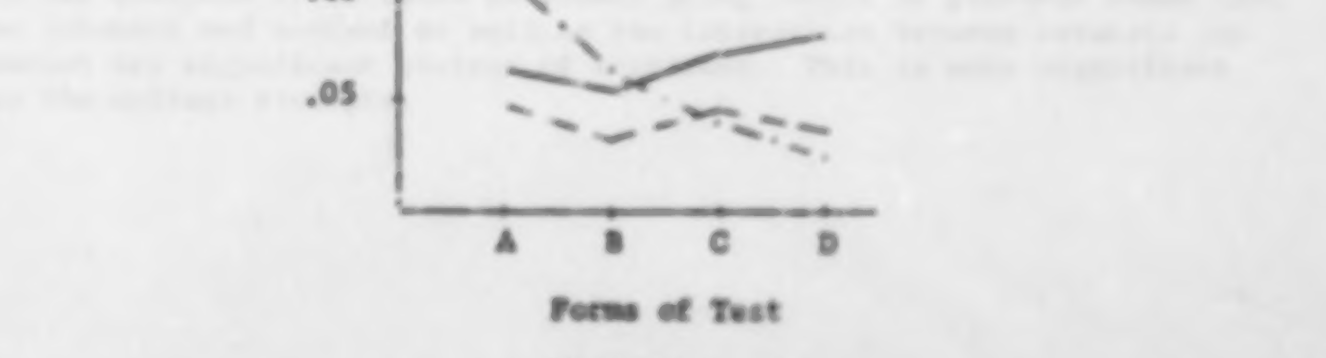

FIGURE 9

MEAN ACCUMULATIVE SCORE (GROUP NORMS) OF COLLEGE SUBJECTS ON PROBLEMS 32, 34, and 36 (TYPE b) FOR THE FOUR FORMS OF EACH PROBLEM.

b) Length of plateaux.

Every experimental subject was also scored using schemata norms. After drawing the performance curves for every subject in every problem of the training sessions, the lengths of plateaux were calculated. Using the length of plateaux for every subject in every problem, analyses of variances were performed separately for high school students and for college students as well as for problems of type a and problems of type b. Tables 7 and 8 present the results of the analyses of variances for the high school students, tables 9 and 10 for the college students.

For the college students the "F" ratio for the main effect schemata, the main effect content, and the interaction between schemata and content are significant at .001 level for problems of type a and for problems of type b.

For the high school students the "F" ratio for the main effect schemata and the main effect content are significant at the .001 level for problems of type a. The "F" ratio for the interaction between schemata and content is not significant.

For problems of type b with the high school students, the "F" ratio for the interaction between schemata and content is significant at the .001 level. The "F" ratio for the main effect schemata is significant at the .05 level while the "F" ratio for the main effect content is not significant.

Figures 10 and 11 present the mean length of plateaux for high school students on problem of type a and b respectively. Looking at these figures it is possible to see that the interaction between schemata and content is highly significant for problems of type b, but not significant for problems of type a.

Figures 12 and 13 present the mean length of plateaux for the college students on problems of type a and b respectively. Inspection of these figures shows that the interaction between schemata and content is highly significant for problems of type a and for problems of type b.

In summary, the results of analyses of variance using group norms and the analyses of variance performed using length of plateaux shows that the schemata and content as well as the interaction between schemata and content are significant sources of variation. This is more significant for the college students.

TABLE 7

ANALYSIS OF VARIANCE FOR PROBLEMS OF TYPE a (TRAINING SESSIONS) ON THE HIGH SCHOOL STUDENTS BASED ON LENGTH OF PLATEAUX

Source	Sum of Squares	df	Variance Estimate	F
Main Effects:				
Schemata	295.061	2	147.500	12.15^{xxx}
Content	112.574	3	37.525	8.71^{xxx}
Subjects	750.333	18	41.685	
Interaction:				
Schemata X Content	50.413	6	8.402	1.75
Schemata X Subjects	437.772	36	12.160	
Content X Subjects	232.509	54	4.306	
Interaction:				
Schemata X Content X Subjects	440.755	108	4.081	
Total	2319.417	227		

xxx
p < .001

TABLE 8

ANALYSIS OF VARIANCE FOR PROBLEMS OF TYPE b (TRAINING SESSIONS) ON THE HIGH SCHOOL STUDENTS BASED ON LENGTH OF PLATEAUX

Source	Sum of Squares	df	Variance Estimate	F
Main Effects:				
Schemata	402.973	2	201.486	4.79^{x}
Content	78.364	3	26.121	2.36
Subjects	1985.535	18	110.306	
Interaction:				
Schemata X Content	271.728	6	45.288	4.31^{xxx}
Schemata X Subjects	1512.858	36	42.023	
Content X Subjects	597.552	54	11.065	
Interaction:				
Schemata X Content X Subjects	1135.108	108	10.510	
Total	5984.118	227		

x
  $p < .05$
xxx
  $p < .001$

TABLE 9

ANALYSIS OF VARIANCE FOR PROBLEMS OF TYPE *a* (TRAINING SESSIONS) ON THE COLLEGE STUDENTS BASED ON LENGTH OF PLATEAUX

Source	Sum of Squares	df	Variance Estimate	F
Main Effects:				
Schemata	188.008	2	94.004	30.73^{xxx}
Content	141.561	3	47.187	19.87^{xxx}
Subjects	271.868	18	15.103	
Interaction:				
Schemata X Content	60.834	6	10.139	4.97^{xxx}
Schemata X Subjects	110.159	36	3.059	
Content X Subjects	128.272	54	2.375	
Interaction:				
Schemata X Content X Subjects	220.333	108	2.040	
Total	1121.035	227		

xxx
$p < .001$

TABLE 10

ANALYSIS OF VARIANCE FOR PROBLEMS OF TYPE $\underline{b}$ (TRAINING SESSIONS) ON THE COLLEGE STUDENTS BASED ON LENGTH OF PLATEAUX

Source	Sum of Squares	df	Variance Estimate	F
Main Effects:				
Schemata	186.061	2	93.030	9.47^{xxx}
Content	261.000	3	87.000	6.67^{xxx}
Subjects	715.710	18	39.761	
Interaction:				
Schemata X Content	630.079	6	105.013	17.53^{xxx}
Schemata X Subjects	353.606	36	9.822	
Content X Subjects	704.500	54	13.046	
Interaction:				
Schemata X Content X Subjects	646.921	108	5.990	
Total	3497.877	227		

^{xxx} $p < .001$

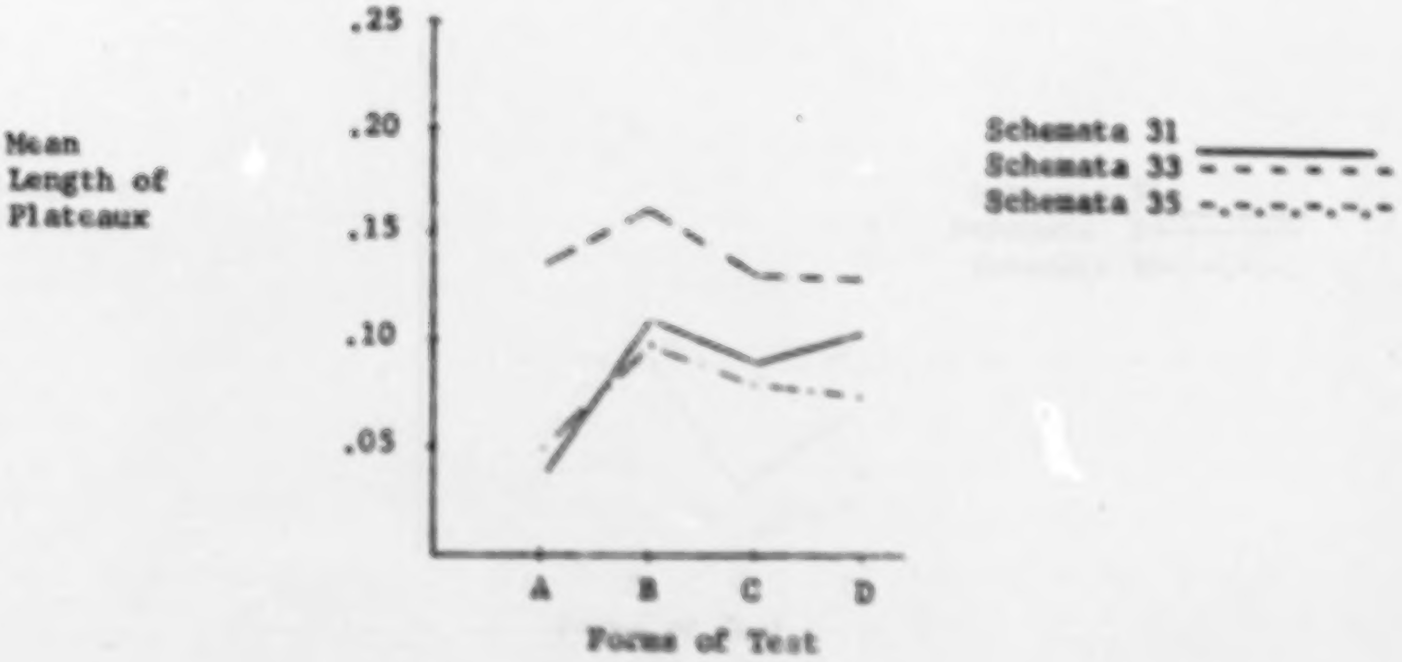

FIGURE 10

MEAN LENGTH OF PLATEAUX OF HIGH SCHOOL SUBJECTS ON PROBLEMS 31, 33, and 35 (TYPE a) FOR THE FOUR FORMS OF EACH PROBLEM.

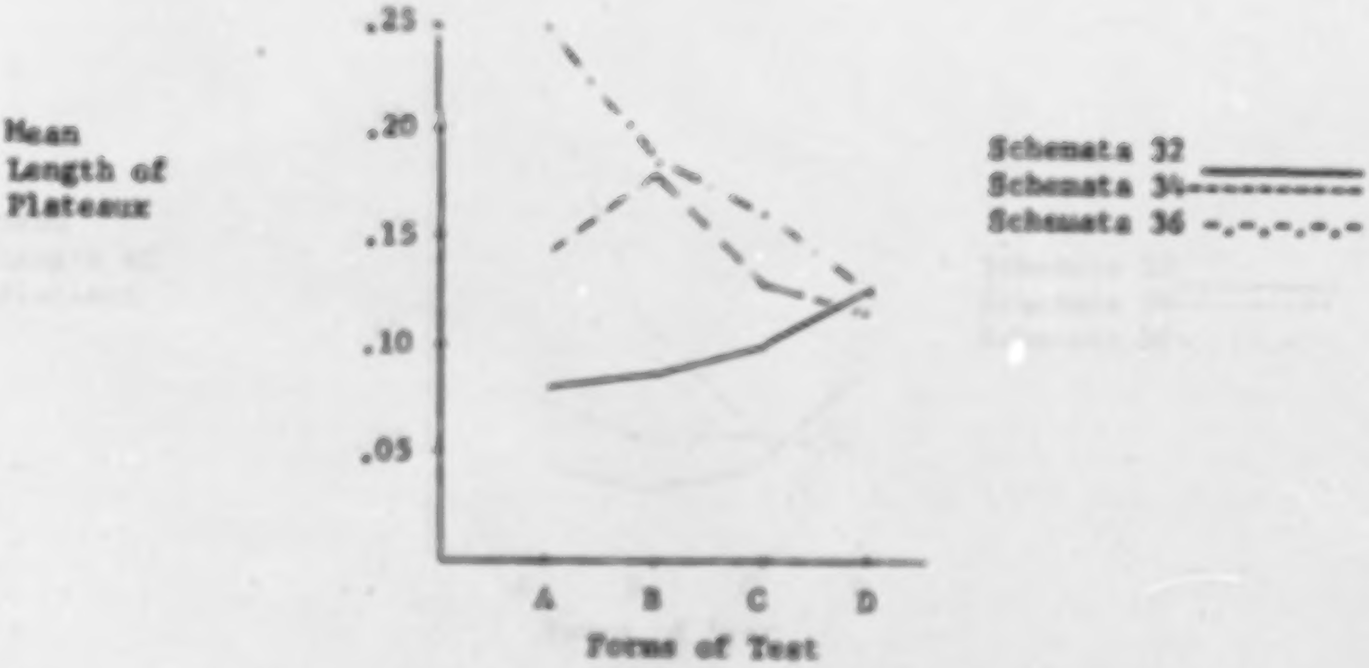

FIGURE 11

MEAN LENGTH OF PLATEAUX OF HIGH SCHOOL SUBJECTS ON PROBLEMS 32, 34, and 36 (TYPE b) FOR THE FOUR FORMS OF EACH PROBLEM.

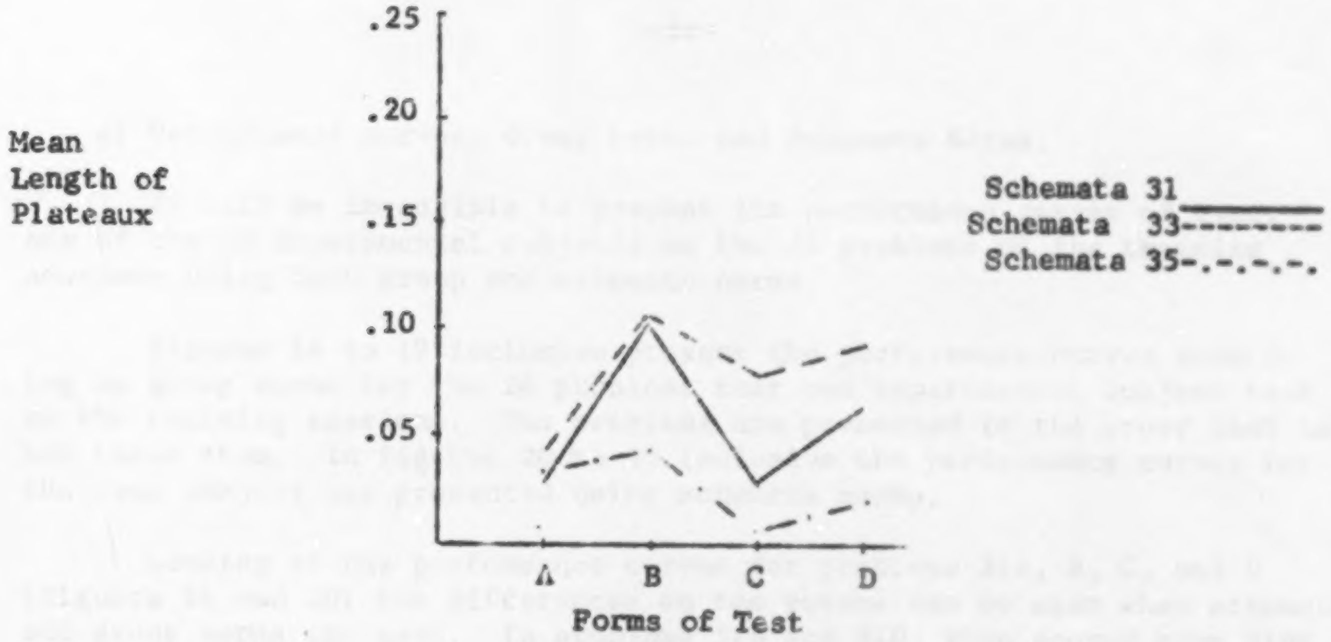

FIGURE 12

MEAN LENGTH OF PLATEAUX OF COLLEGE SUBJECTS ON PROBLEMS 31, 33, and 35 (TYPE a) FOR THE FOUR FORMS OF EACH PROBLEM.

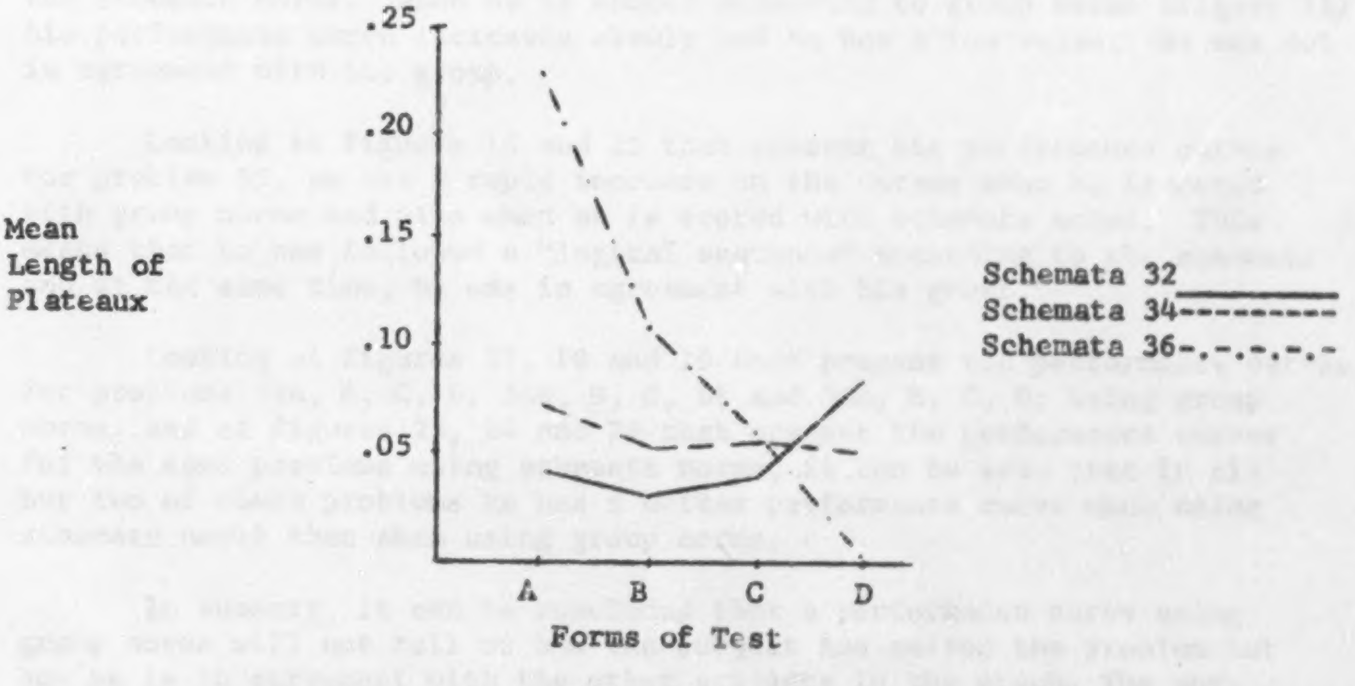

FIGURE 13

MEAN LENGTH OF PLATEAUX OF COLLEGE SUBJECTS ON PROBLEMS 32, 34, and 36 (TYPE b) FOR THE FOUR FORMS OF EACH PROBLEM.

c) Performance curves, Group norms and Schemata Norms.

It will be impossible to present the performance curves of every one of the 38 experimental subjects on the 24 problems of the training sessions using both group and schemata norms.

Figures 14 to 19 inclusive present the performance curves according to group norms for the 24 problems that one experimental subject took on the training sessions. The problems are presented in the order that he had taken them. In figures 20 to 25 inclusive the performance curves for the same subject are presented using schemata norms.

Looking at the performance curves for problems 31A, B, C, and D (figures 14 and 20) the differences on the curves can be seen when schemata and group norms are used. In problems 31B and 31D, when scored according to the group norms, the subject received a very high score, his performance curve rapidly increases (figure 14). This means that he was in agreement with the group. Nevertheless, when his performance is scored according to schemata norms, the curves show plateau and very low values. He did not follow any "logical sequence" as defined by the schemata.

In problem 33C (figure 21) the subject shows a good performance according to schemata norms, his curve is increasing rapidly and no plateau is observed. He has followed one "logical sequence" as defined by the schemata norms. When he is scored according to group norms (figure 15) his performance curve increases slowly and he has a low value. He was not in agreement with his group.

Looking at figures 16 and 22 that present the performance curves for problem 35, we see a rapid increase on the curves when he is scored with group norms and also when he is scored with schemata norms. This means that he has followed a "logical sequence" according to the schemata and at the same time, he was in agreement with his group.

Looking at figures 17, 18 and 19 that present the performance curves for problems 32A, B, C, D; 34A, B, C, D; and 36A, B, C, D; using group norms, and at figures 23, 24 and 25 that present the performance curves for the same problems using schemata norms, it can be seen that in all but two of these problems he has a better performance curve when using schemata norms than when using group norms.

In summary, it can be concluded that a performance curve using group norms will not tell us how the subject has solved the problem but how he is in agreement with the other subjects in the group. The performance curve using schemata norms will tell us how the subject has approached the problem. If he has used a "logical sequence", no plateaux will appear on the performance curve and his score and performance curve will be the same regardless of the group to which he belongs.

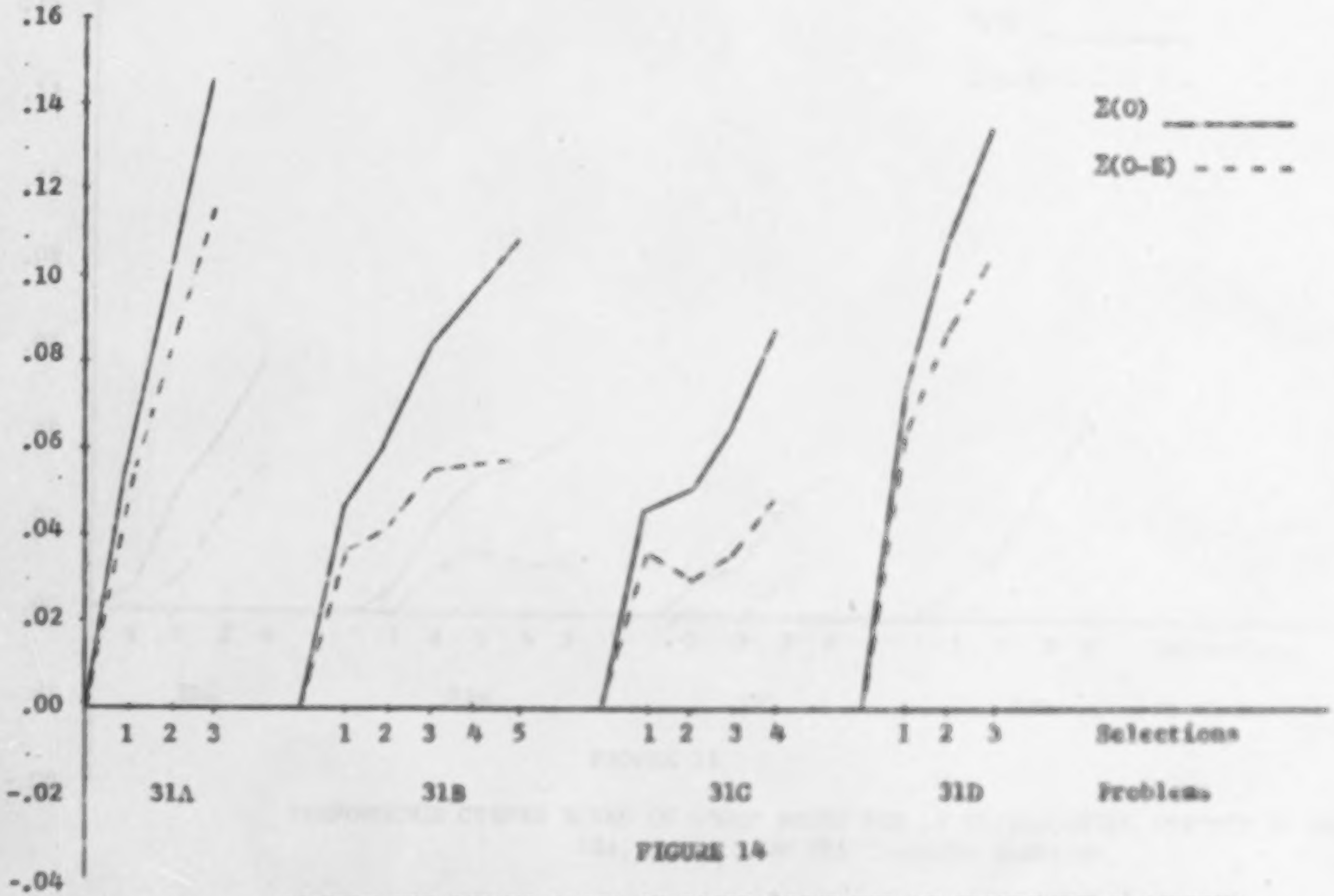

FIGURE 14

PERFORMANCE CURVES BASED ON GROUP NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 31A, B, C, D OF THE TRAINING SESSIONS.

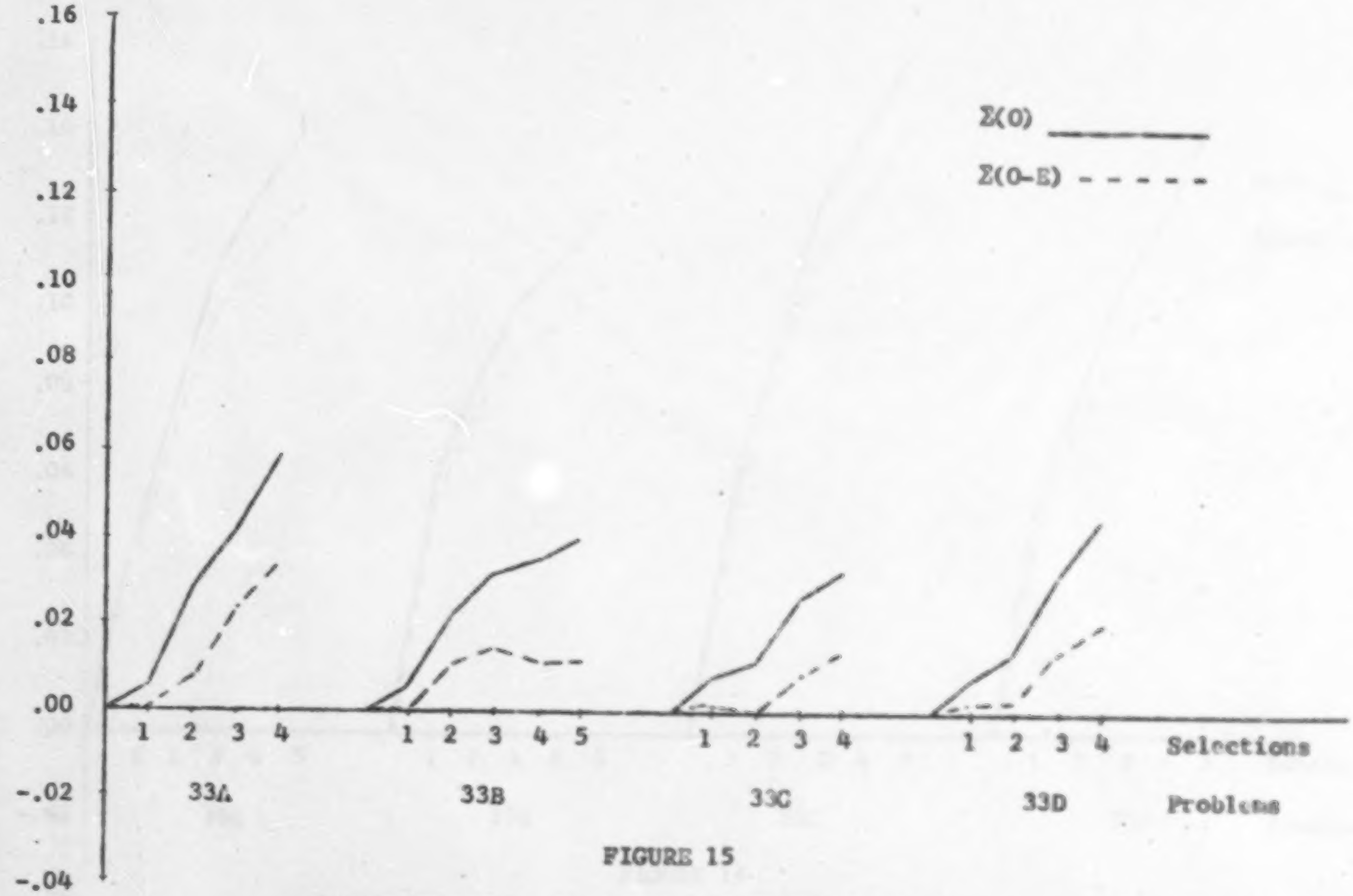

FIGURE 15

PERFORMANCE CURVES BASED ON GROUP NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 33A, B, C, D OF THE TRAINING SESSIONS.

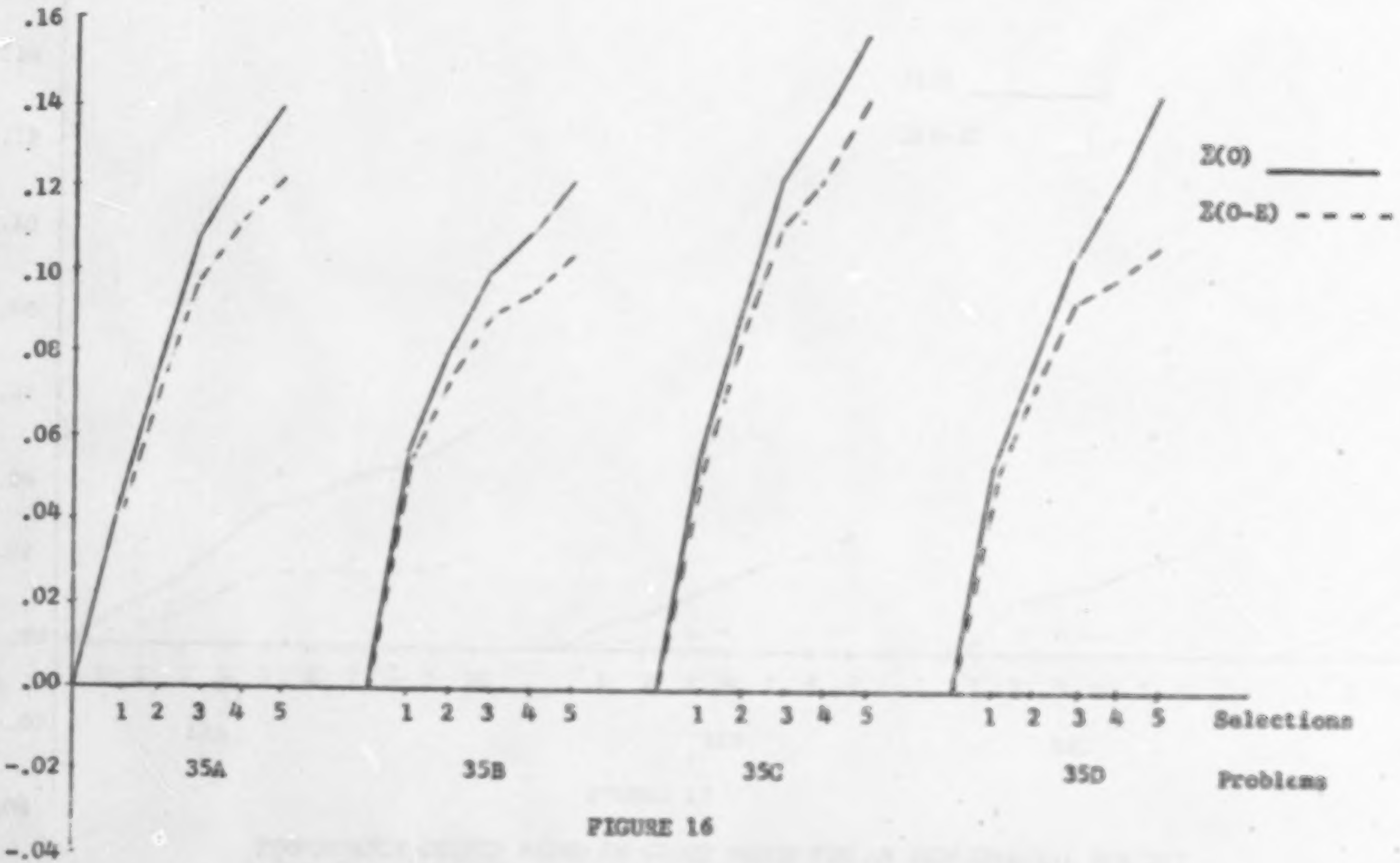

FIGURE 16

PERFORMANCE CURVES BASED ON GROUP NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 35A, B, C, D OF THE TRAINING SESSIONS.

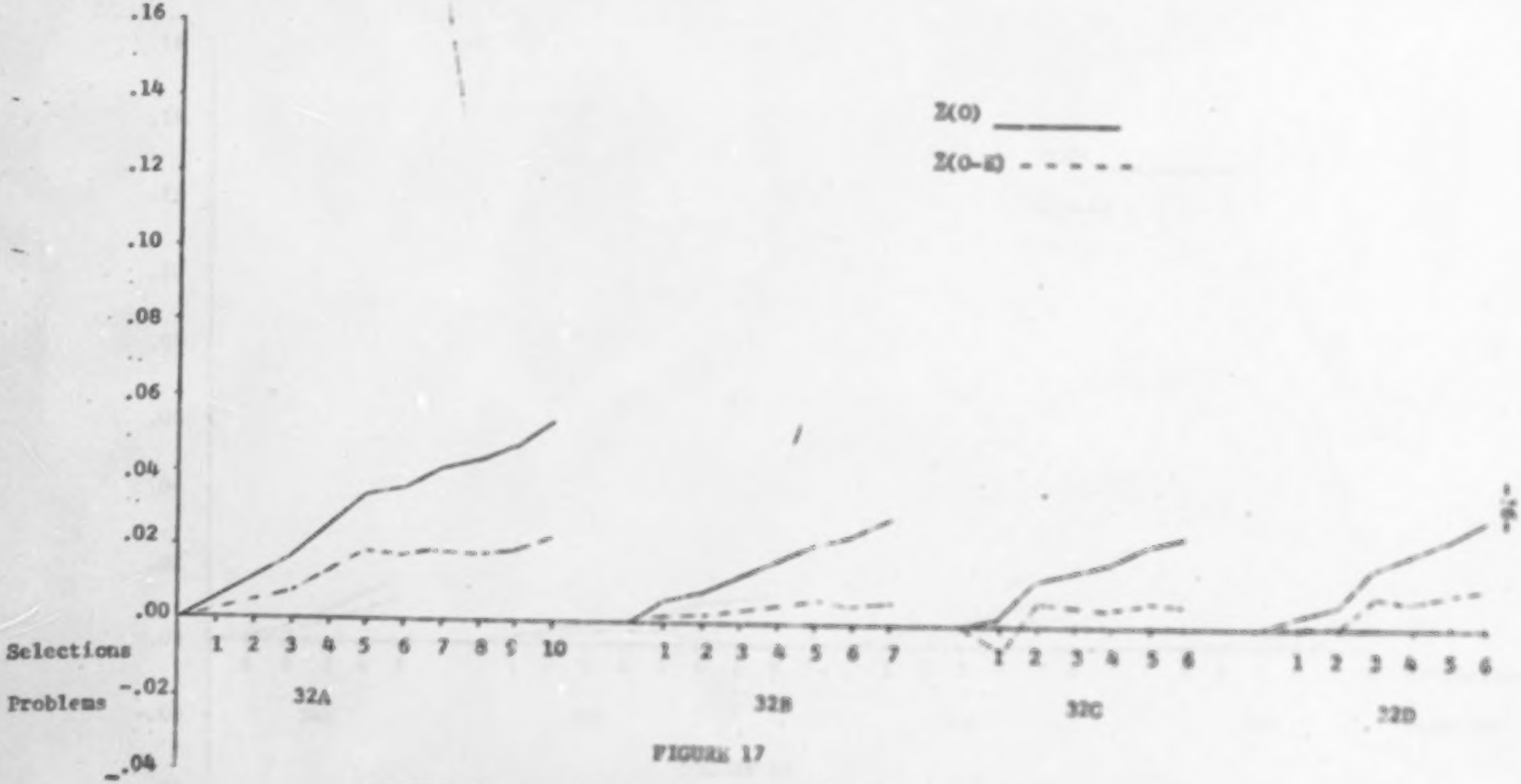

FIGURE 17

PERFORMANCE CURVES BASED ON GROUP NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 32A, B, C, D OF THE TRAINING SESSIONS.

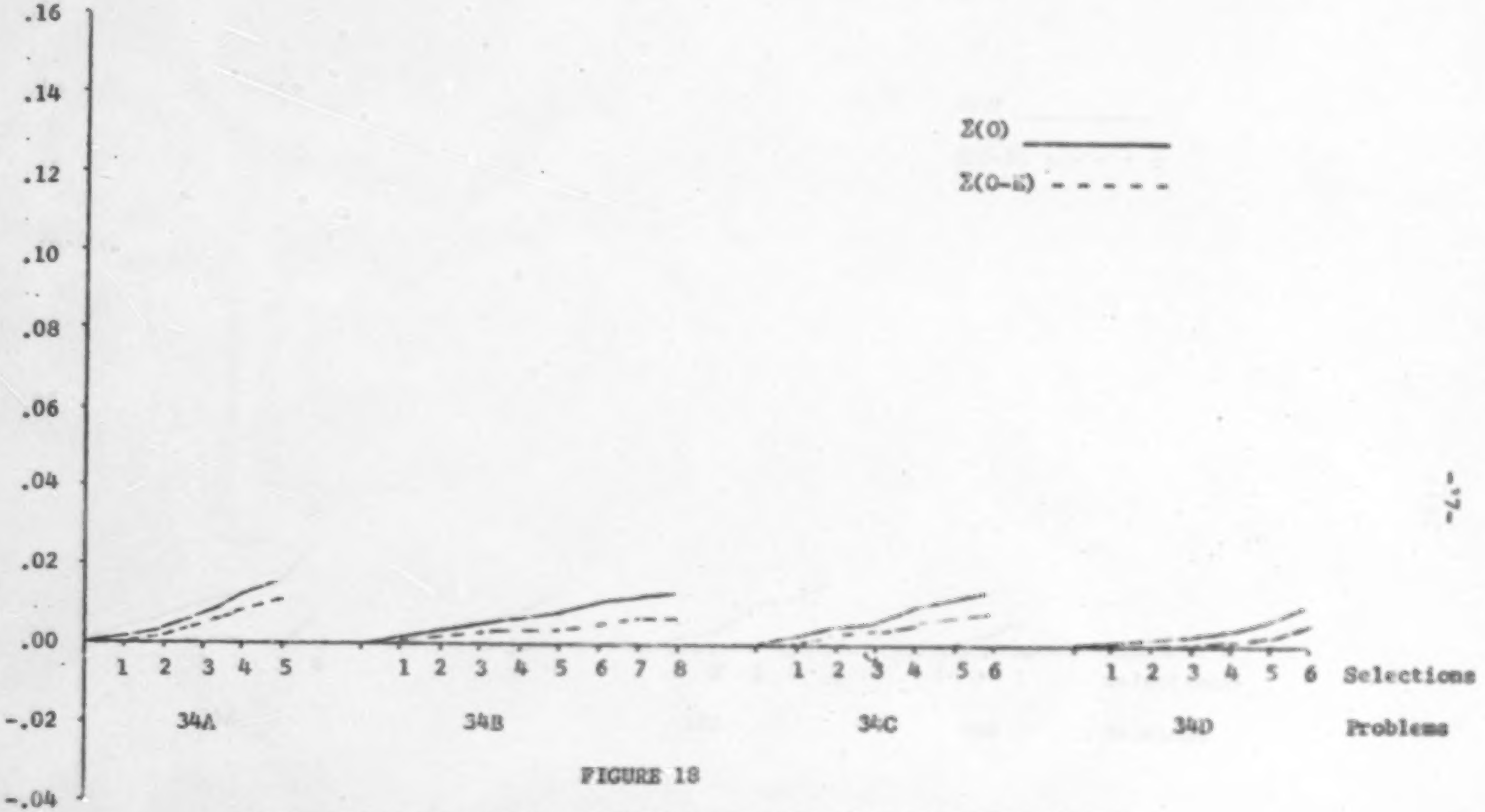

FIGURE 18

PERFORMANCE CURVES BASED ON GROUP NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 34A, B, C, D OF THE TRAINING SESSIONS.

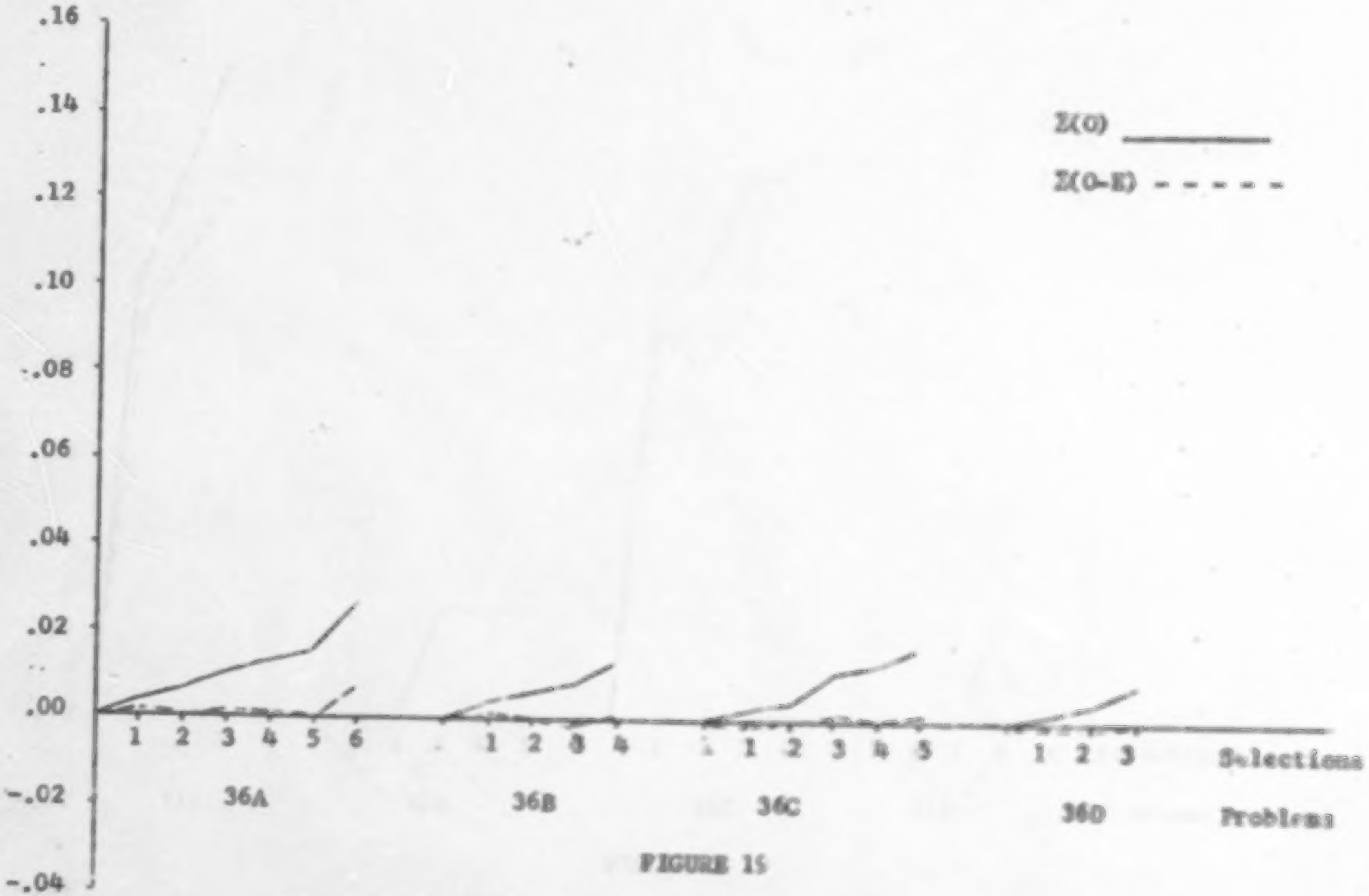

FIGURE 15

PERFORMANCE CURVES BASED ON GROUP NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 36A, B, C, D OF THE TRAINING SESSIONS.

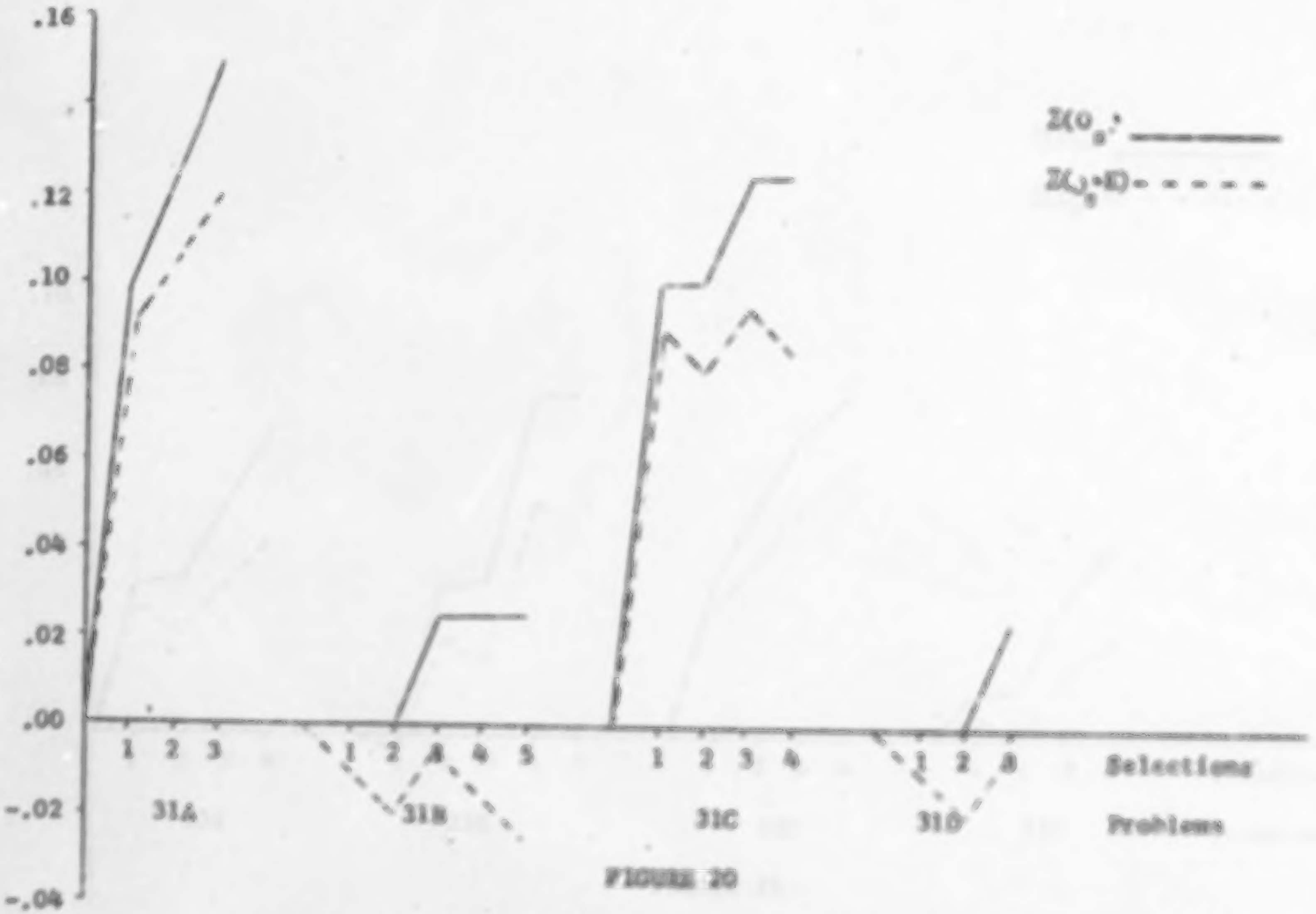

FIGURE 20

PERFORMANCE CURVES BASED ON SCHEMATA NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 31A, B, C, D OF THE TRAINING SESSIONS.

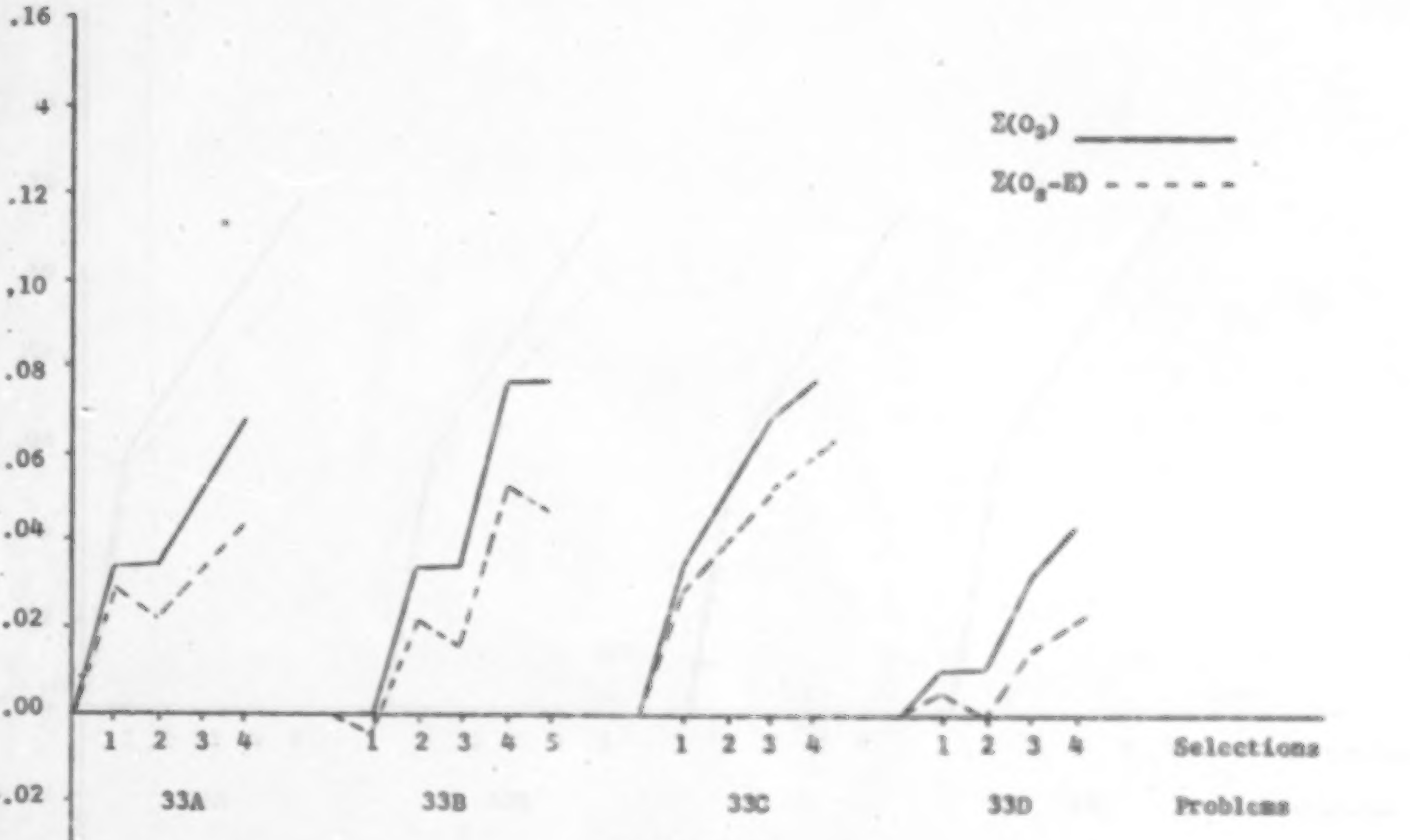

FIGURE 21

PERFORMANCE CURVES BASED ON SCHEMATA NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 33A, B, C, D OF THE TRAINING SESSIONS.

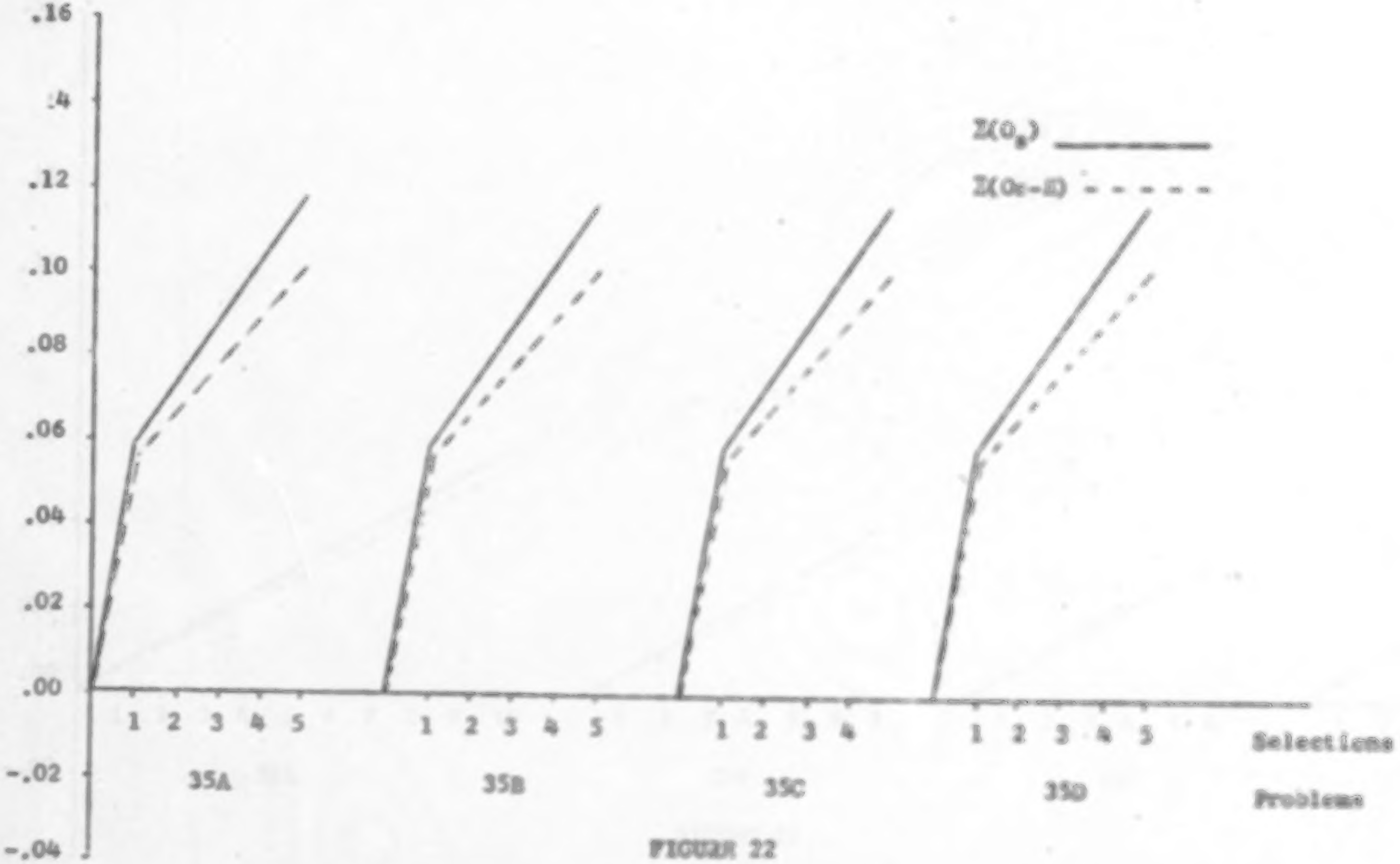

FIGURE 22

PERFORMANCE CURVES BASED ON SCHEMATA NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 35A, B, C, D OF THE TRAINING SESSIONS.

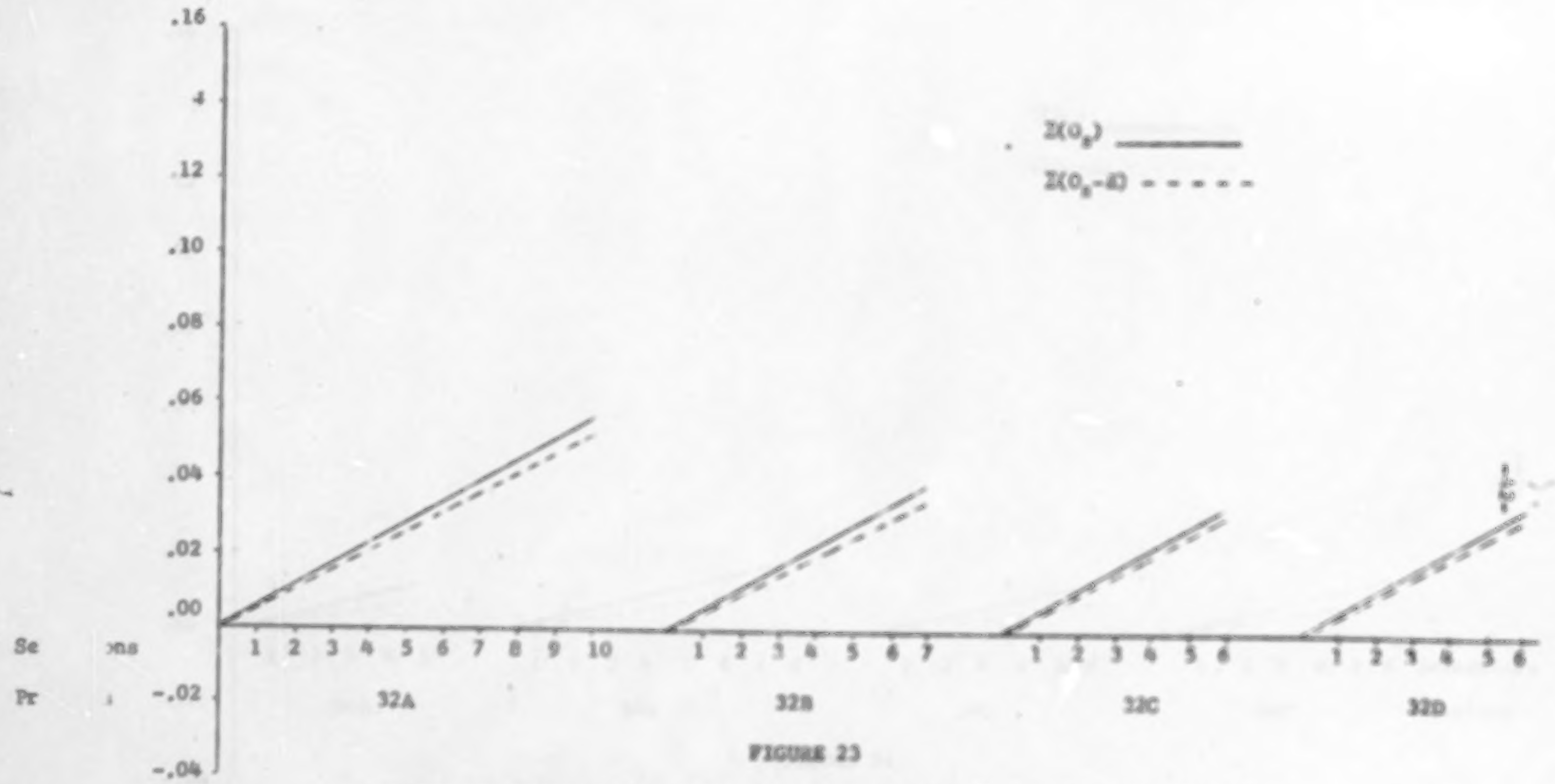

FIGURE 23

PERFORMANCE CURVES BASED ON SCHEMATA NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 32A, B, C, D OF THE TRAINING SESSIONS.

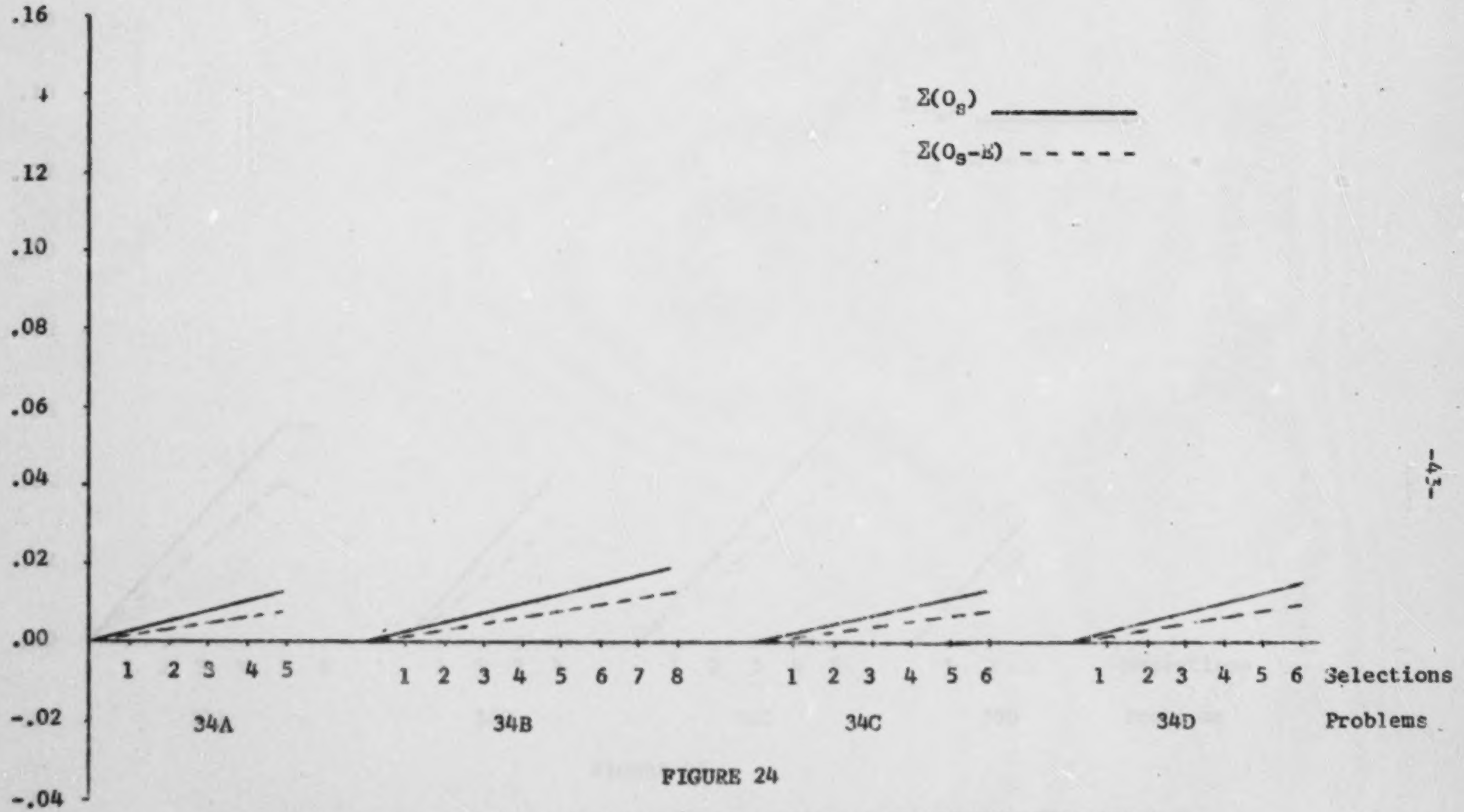

FIGURE 24

PERFORMANCE CURVES BASED ON SCHEMATA NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 34A, B, C, D OF THE TRAINING SESSIONS.

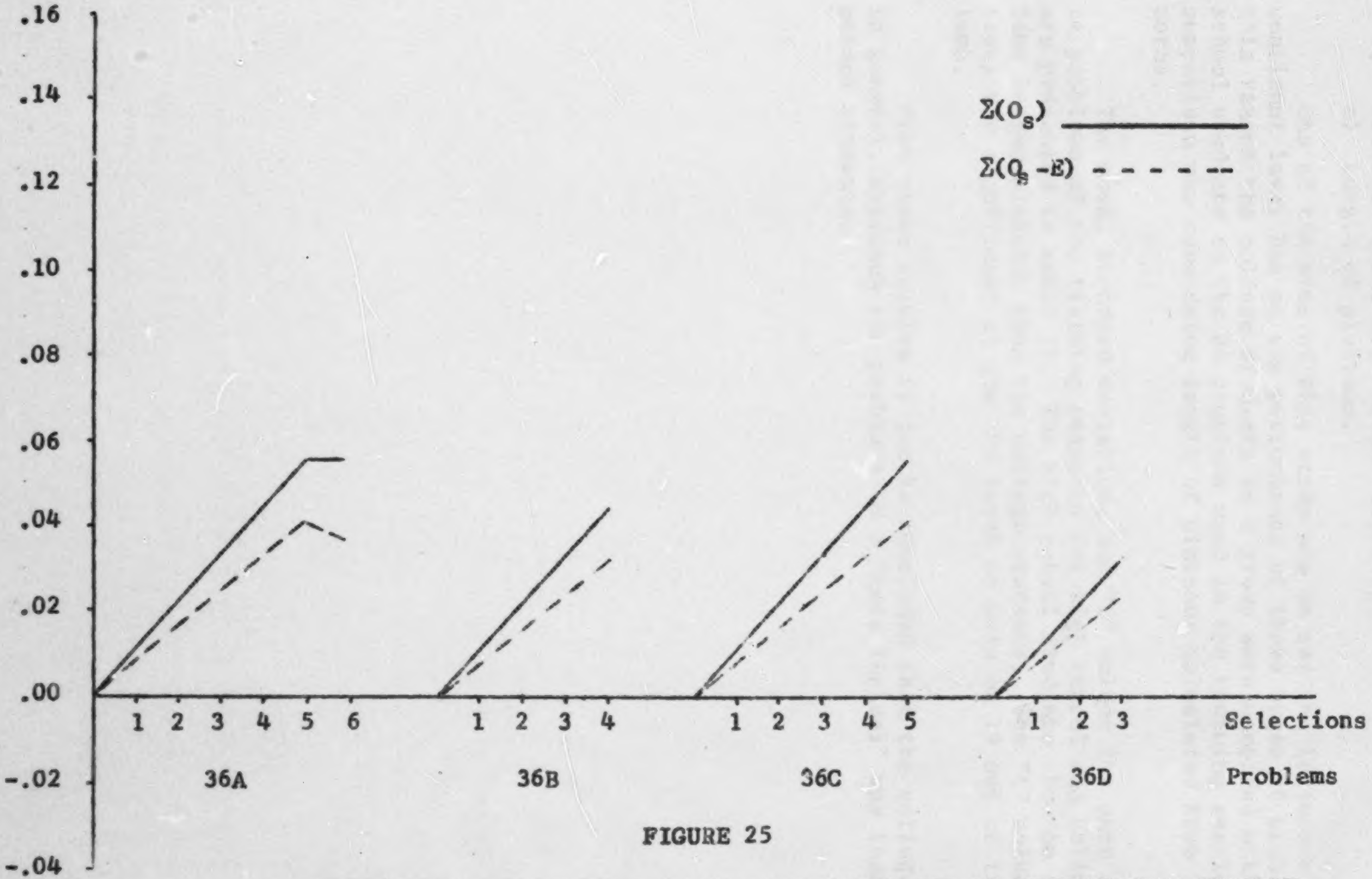

FIGURE 25

PERFORMANCE CURVES BASED ON SCHEMATA NORMS FOR AN EXPERIMENTAL SUBJECT ON PROBLEMS 36A, B, C, D OF THE TRAINING SESSIONS.

2. College versus high school students.
   a) Length of plateaux.

One of the aims of this study was to see the influence that educational level has on the performance of these types of problems. For this reason the college students as a group were compared with the high school students on the 24 problems used in the training sessions. This comparison was done using length of plateaux calculated from the schemata norms.

The mean, standard deviation, and "t" values for each one of the 24 problems of the training sessions for high school and college students are presented in table 11. The high school students show on all the problems longer plateaux than the college students. The "t" values (one tail test) are significant at the .05 level or more on 19 out of the 24 problems.

From these results it can be concluded that the college students, in general, approach the problems in a "more logical" way than the high school students.

TABLE 11

MEAN, AND STANDARD DEVIATIONS OF LENGTH OF PLATEAUX, NUMBER OF SUBJECTS AND "t" VALUES FOR EACH ONE OF THE 24 PROBLEMS OF THE TRAINING SESSIONS FOR THE EXPERIMENTAL SUBJECTS IN HIGH SCHOOL AND COLLEGE.

Problems		High School			College			
		M	σ	N	M	σ	N	"t" Values
	31A	1.42	1.70	19	1.05	1.10	19	.80
	B	4.42	2.21	19	4.11	1.97	19	.46
	C	3.74	2.71	19	1.16	1.39	19	3.69***
	D	4.11	2.12	19	2.68	.86	19	2.71**
	33A	5.58	2.66	19	1.79	1.54	19	5.37***
Type a	B	6.53	2.37	19	4.26	2.20	19	3.06**
	C	5.32	2.97	19	3.32	2.62	19	2.20*
	D	5.32	2.49	19	3.89	1.89	19	1.99*
	35A	2.00	2.43	19	1.21	2.07	19	1.08
	B	3.95	4.39	19	1.74	2.12	19	1.98*
	C	3.37	3.63	19	.47	1.39	19	3.25**
	D	3.26	3.49	19	.95	1.54	19	2.64**
	32A	3.37	2.81	19	1.89	1.84	19	1.99*
	B	3.53	3.33	19	1.37	2.01	19	2.42*
	C	4.05	2.98	19	1.79	2.28	19	2.63**
	D	5.11	3.21	19	3.53	3.14	19	1.53
	34A	5.89	6.40	19	3.00	3.74	19	1.70*
Type b	B	7.32	5.78	19	2.21	3.07	19	3.40**
	C	5.26	5.98	19	2.26	2.83	19	1.98*
	D	4.79	4.54	19	2.16	2.76	19	2.16*
	36A	10.00	4.33	19	9.26	4.83	19	.50
	B	7.47	5.08	19	4.53	4.74	19	1.84*
	C	6.53	5.58	19	2.63	4.43	19	2.39*
	D	5.05	5.54	19	.26	.71	19	3.74***

*P<.05
**P<.01
***P<.001

B. Pre and post-testing sessions.

1. Control versus experimental subjects

a) Schemata norms:

Table 12 presents the mean differences, the standard deviations of differences, and "t" values for differences between experimental and control subjects on the accumulative score for every problem administered during the pre and post-testing sessions for the high school and college students.

Comparing experimental and control subjects on the accumulative score (schemata norms) in problems 1, 19, and 25 which were administered in the pre and post-testing sessions, no significant differences are found neither for high school nor for college students. It is possible that memory has had an influence on the performance of the experimental subjects in the second administration of these problems. For here it appears that memory of the first administration has overcome the effect of the training between the administrations.

Problems 31D' and 35B' have the same schemata and content as the one used in the training sessions. The differences between control and experimental subjects are significant at the .001 level for problem 31D' with both the high school and college students; and at the .01 level for problem 35B' with college students. For problem 35B' there is no significant difference between control and experimental high school subjects.

In problem 32F there is no significant difference between control and experimental subjects. The content of problem 32F was not similar to the one used in the training sessions. Problem 36F has the same schemata and content as the one used in the training sessions; the M_D of the accumulative score is significant at the .001 level for college students and at .01 level for high school students.

Problems 26 and 41A were new problems with different schemata than the ones used in the training sessions. There is no significant difference between control and experimental high school and college students.

In summary, these results seem to indicate that when the problems have the same schemata and content as the ones used in the training sessions, the differences between experimental and control subjects on the "logical" way of approaching a problem are significant. But, introducing a change in the schemata or in the content, the subjects with training seem to approach the problem in as similar a manner as the subjects without training do when judged by the accumulative score obtained according to the schemata norms.

TABLE 12

MEAN DIFFERENCES, STANDARD DEVIATIONS OF DIFFERENCES BETWEEN EXPERIMENTAL AND CONTROL SUBJECTS ON THE ACCUMULATIVE SCORES (SCHEMATA NORMS), NUMBER OF SUBJECTS AND "t" VALUES FOR EACH ONE OF THE PROBLEMS OF THE PRE-TESTING AND POST-TESTING SESSIONS OF HIGH SCHOOL AND COLLEGE STUDENTS.

		High School				College			
Problems		M_D	σ_D	N	"t"	M_D	σ_D	N	"t"
Pre-Testing	1	-.00413	.01774	19	-1.01	-.00061	.02572	19	-.10
	19	.00754	.02673	19	1.23	-.00666	.02475	19	-1.17
	25	.00371	.02239	19	.72	.00556	.02055	19	1.18
Post-Testing	1	.00255	.02676	19	.42	.00061	.01648	19	.16
	19	.00710	.02963	19	1.04	.00380	.00804	19	.47
	25	-.00657	.02357	19	-1.21	.00101	.02303	19	.19
Type a	31D'	.04868	.05224	19	4.06***	.05921	.06294	19	4.10***
	35B'	.01779	.04901	19	1.58	.03326	.04953	19	2.93**
Type b	32F	-.00010	.00082	19	-.53	-.00002	.00068	19	-.11
	36F	.00280	.00375	19	3.26**	.00422	.00265	19	6.94***
New Problems	26	.01053	.03969	19	1.16	-.00203	.04742	19	-.18
	41A	-.00876	.03787	19	-1.01	-.00643	.05007	19	-.56

*$P<.05$
**$P<.01$
***$P<.001$

b) Length of plateaux:

By inspection of the performance curves using schemata norms (figures 20 to 33 inclusive), it can be seen that there are moments in the solution of the problem when no improvement is observed. This means that the subject had selected a useless question, a question that has a score of zero. Observing the performance curves of every subject in all the problems, it is possible to know the length of plateaux that each subject has in every problem. In table 13 the mean differences, standard deviation of differences, number of subjects, and "t" values are given for the differences between control and experimental high school and college subjects for the problems of the pre and post-testing sessions.

When comparing experimental and control subjects in problems 1, 19, and 25 that were used in the pre and post-testing sessions, no differences on the length of plateaux is observed.

Problems 31D' and 35B' have the same schemata and content as the one used in the training sessions; the differences between control and experimental are significant at the .01 level or more.

Problem 32F has the same figure but different content than the one used in the training sessions; the differences are not significant. Problem 36F has the same schemata and content as the one used in the training sessions; the differences between control and experimental are significant at .001 level for the college students and at the .01 level for the high school students.

Problems 26 and 41A were new problems. The differences are in the expected direction, but a .05 level of significance was reached on problem 26 only with the college students.

In summary, by the study of length of plateaux comparing control and experimental subjects, it can be concluded that when control and experimental subjects know the problem there is no significant difference in the way that they approach the problem. But, there is a significant difference when the problem has the same schemata and content as the one used in the training sessions. When the problems have a different schemata or content than the one used in the training sessions, the subjects with training always approach the problem in a more "logical" way than the subjects without training, nevertheless the differences do not always reach a level of significance.

Comparing these results with the conclusions on accumulative score, it can be seen that, in general, they are similar. Nevertheless, the study of length of plateaux seems to be a more sensitive technique than the study of the accumulative score. The accumulative score is obtained by accumulating the score corresponding to every question that the subject has asked. If the subject asked useless questions, he received a score of zero; yet he is not punished by the number of useless questions he asked. However, the useless questions are taken into consideration in the study of length of plateaux.

TABLE 13

MEAN DIFFERENCES, STANDARD DEVIATIONS OF DIFFERENCES BETWEEN CONTROL AND EXPERIMENTAL SUBJECTS ON THE LENGTH OF PLATEAUX, NUMBER OF SUBJECTS AND "t" VALUES FOR EACH ONE OF THE PROBLEMS OF THE PRE-TESTING AND POST-TESTING SESSIONS OF HIGH SCHOOL AND COLLEGE STUDENTS.

PROBLEMS		High School				College			
		M_D	σ_D	N	"t"	M_D	σ_D	N	"t"
Pre-testing	1	.36842	2.27597	19	.71	-.10526	1.37246	19	-.33
	19	.63158	3.32788	19	.83	.36842	2.47536	19	.65
	25	1.05263	5.70755	19	.80	1.63158	4.01589	19	1.77
Post testing	1	-.10526	1.99723	19	-.23	-.05263	.60469	19	-.38
	19	-.15789	3.82874	19	-.18	.63158	2.71856	19	1.01
	25	-.84210	3.61667	19	-1.01	.36842	1.92519	19	.83
type a	31D'	1.73684	2.57196	19	2.94**	2.42105	2.43474	19	4.33***
	35B'	2.63158	3.75865	19	3.05**	3.31579	4.67977	19	3.09**
type b	32F	.57895	3.99099	19	.63	.31579	3.22902	19	.43
	36F	4.89474	7.45438	19	2.86**	8.15579	5.14284	19	6.91***
New Problems	26	.68421	2.86637	19	1.04	1.36842	2.67956	19	2.23*
	41A	.78947	2.14179	19	1.61	1.05263	3.03443	19	1.51

*$P<.05$
**$P<.01$
***$P<.001$

c) Performance curves:

It is impossible to present the performance curves for every subject on all the problems they have taken. The performance curves for a control-experimental pair (schemata norms) are presented in figures 26 to 33 inclusive. This pair was selected not because it emphasized the differences between control and experimental subject, but, because, according to the author, it is one of the typical cases.

The performance curves for problem 1, pre and post-testing, for the control-experimental pair presented in figure 26 show a plateau on the curve of the control subject in the post-testing sessions.

In figures 27 and 28 the performance curves for problem 19 in the pre- and post-testing sessions show higher values for the experimental subject than for the control subject and no plateaux on the curve of the experimental subject on the post-testing session.

In the performance curves of problem 25 (figure 29) the experimental subject shows a higher value on the pre-testing session but longer plateaux than the control subject. The performance curve for the control subject shows no plateaux and higher values than the performance curve of the experimental subject in the post-testing session.

The performance curves for problems 31D' and 35B' (figure 30) show a "better" performance for the experimental than for the control subject. The experimental subject has higher values and no plateaux at all, while the control subject has lower values and longer plateaux.

Looking at figures 31 and 32 for the performance curves in problems 32F and 36F, the experimental subject shows no plateaux at all, while the control subject shows 6 and 12 plateaux respectively which are the maximum possible length of plateaux for these two problems.

Figure 33 shows higher value and longer length of plateaux for the control than for the experimental subject on problem 26. In problem 41A the experimental subject reached a higher value but also shows a plateaux on the performance curve.

In summary, the performance curves of the control-experimental pair presented here show no clear differentiation between the two subjects on the performance curves of problems 1, 19, and 25 in the pre-testing sessions. When the same problems 1, 19, and 25 were administered at the end of the experiment, the experimental subjects had a "better" performance on problems 1 and 19. In problem 25 the control subject had a "better" performance than the experimental subject.

Looking at the performance curves for problems 31D',35B', 32F and 36F, a clear differentiation is demonstrated. The experimental subject has no plateaux at all on the performance curves. This means that he has solved the problems using a "logical" sequence of questions. The control subject showed a large number of plateaux on the performance curves of these problems. This means that he has solved the problems using a "non-logical" sequence according to the schemata norms. Regarding the new problems 26 and 41A no clear differentiation between the performance of the two subjects is found.

The performance curves of just one control-experimental pair was selected among the 38 control-experimental pairs. It is not possible to say that the performance curves of all the control-experimental subjects are like the ones presented here; but, in general, they follow the trend explained above.

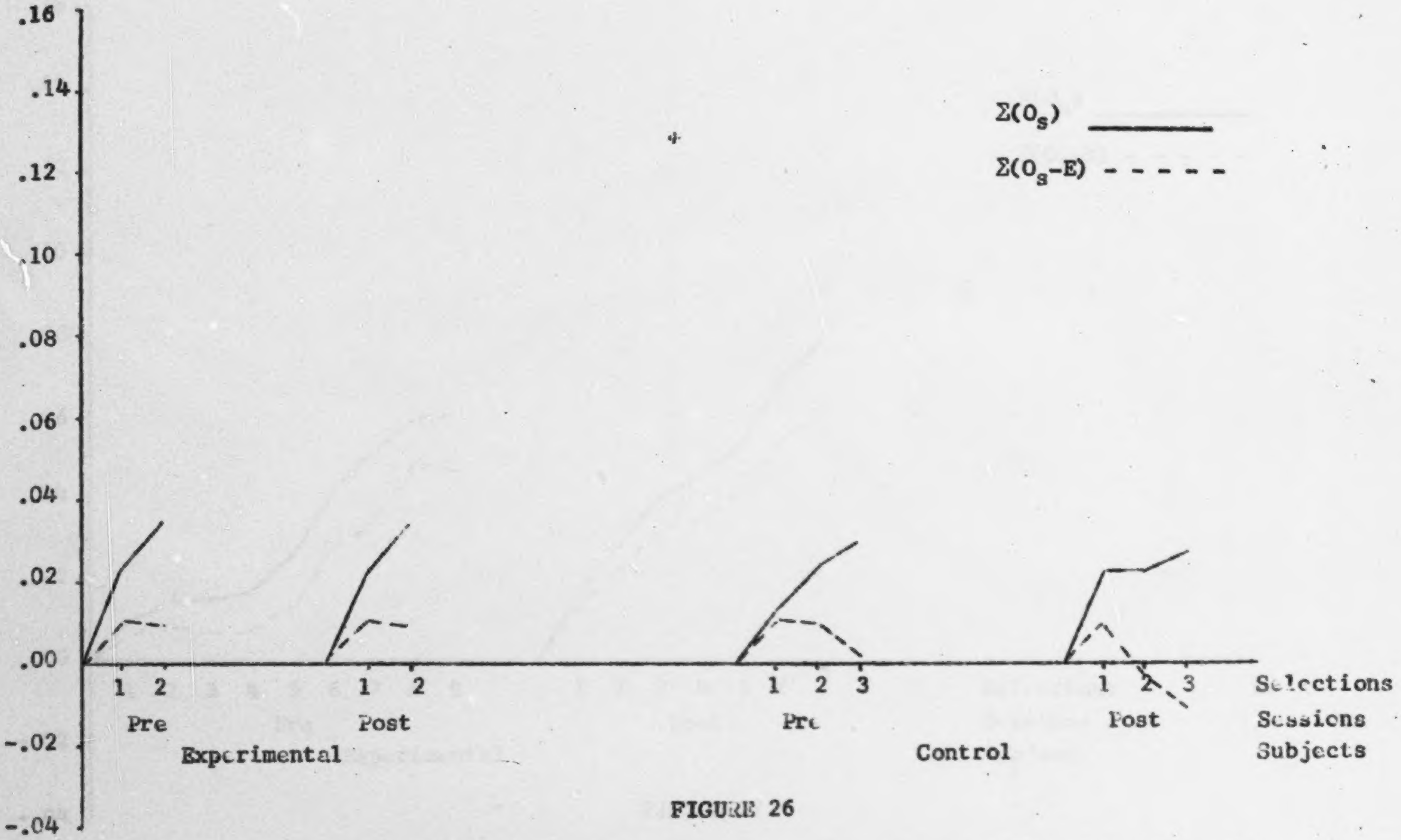

FIGURE 26

PERFORMANCE CURVES FOR A CONTROL-EXPERIMENTAL PAIR (SCHEMATA NORMS) FOR PROBLEM 1 PRE- and POST-TESTING SESSIONS.

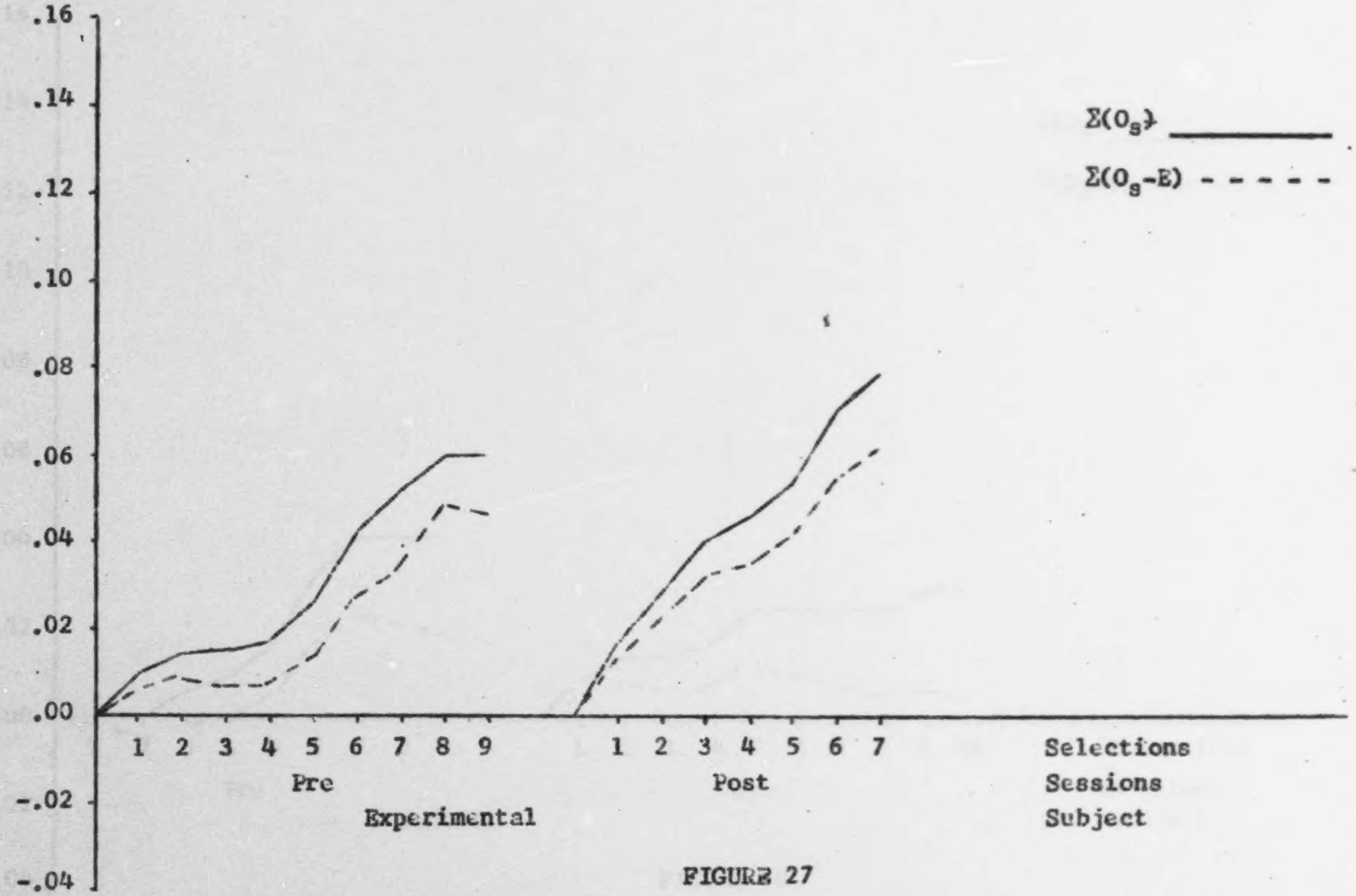

FIGURE 27

PERFORMANCE CURVES FOR A EXPERIMENTAL SUBJECT (SCHEMATA NORMS) FOR PROBLEM 19 PRE AND POST-TESTING SESSIONS.

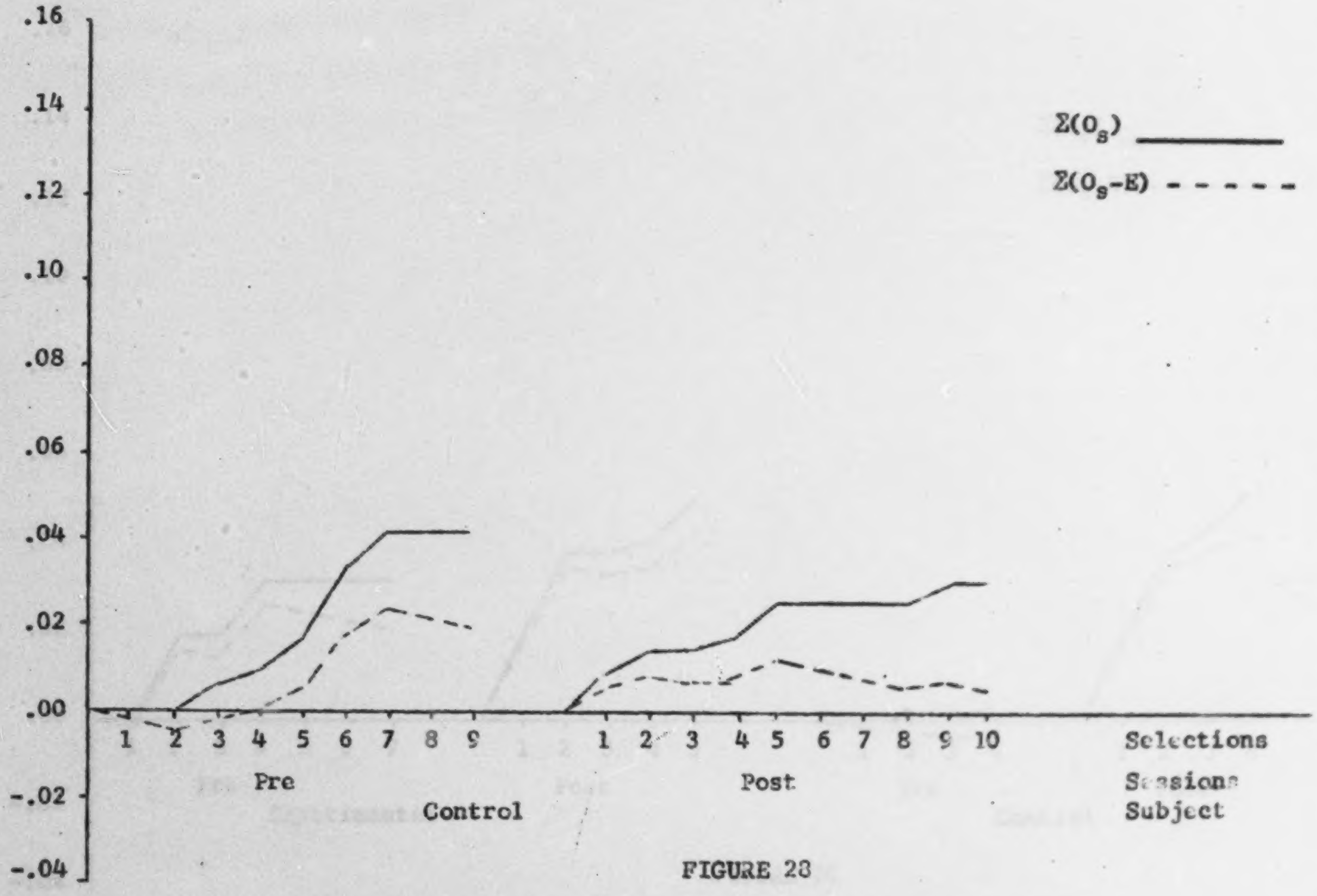

FIGURE 28

PERFORMANCE CURVES FOR A CONTROL SUBJECT (SCHEMATA NORMS) FOR PROBLEM 19 PRE AND POST-TESTING SESSIONS.

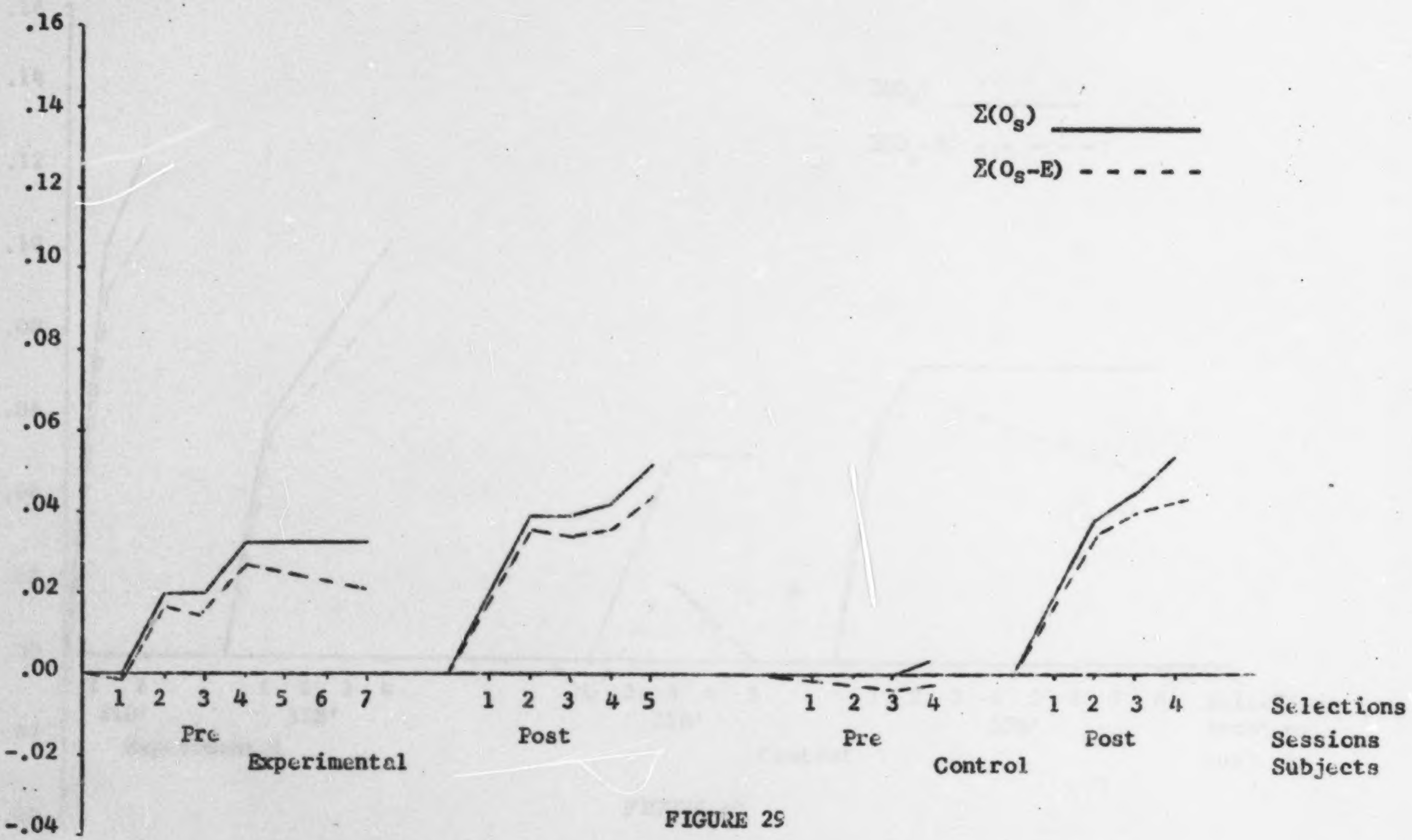

FIGURE 29

PERFORMANCE CURVES FOR A CONTROL-EXPERIMENTAL PAIR (SCHEMATA NORMS) FOR PROBLEM 25 PRE AND POST-TESTING SESSIONS.

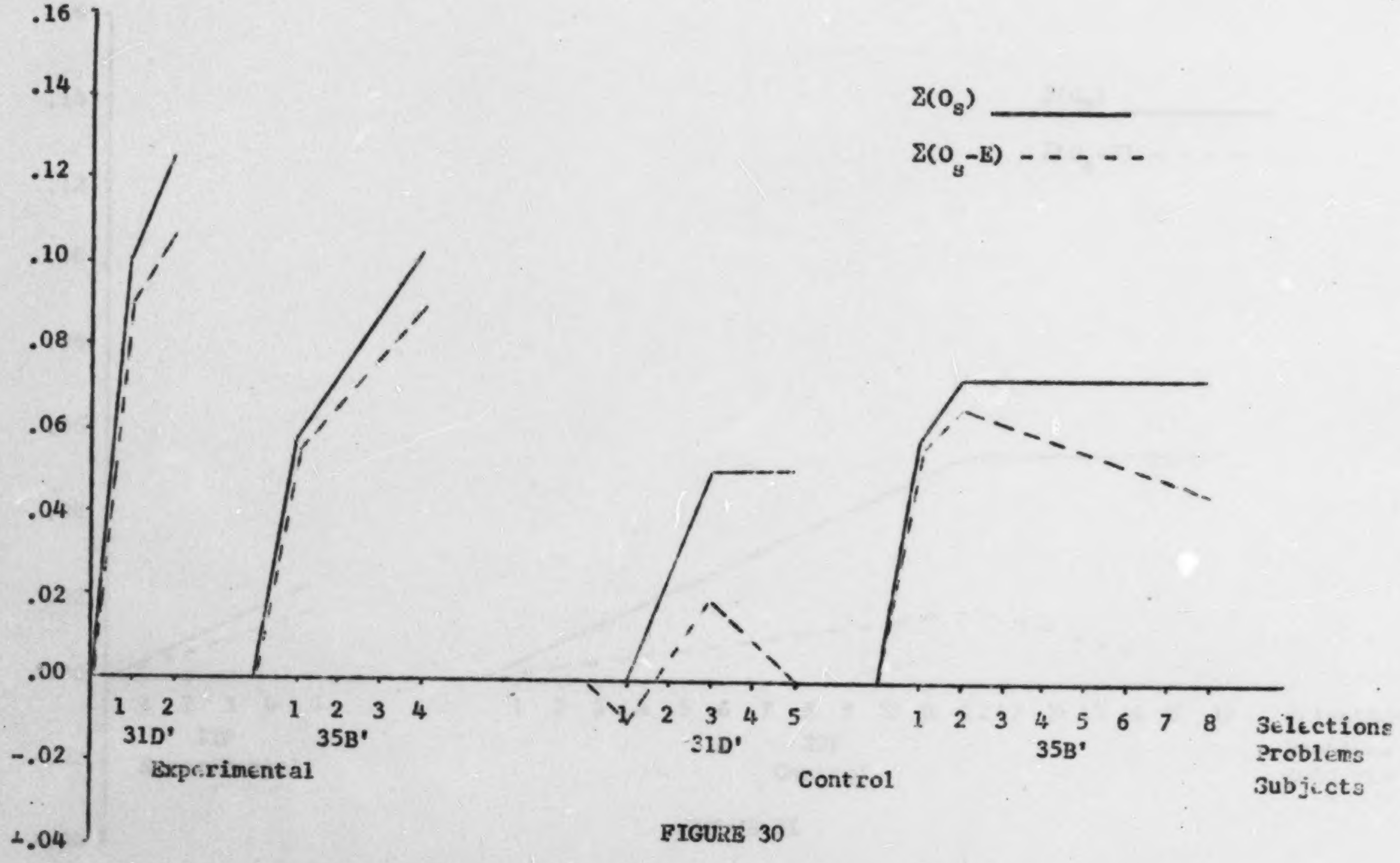

FIGURE 30

PERFORMANCE CURVES FOR A CONTROL-EXPERIMENTAL PAIR (SCHEMATA NORMS)
FOR PROBLEMS 31D' and 35B' POST-TESTING SESSIONS.

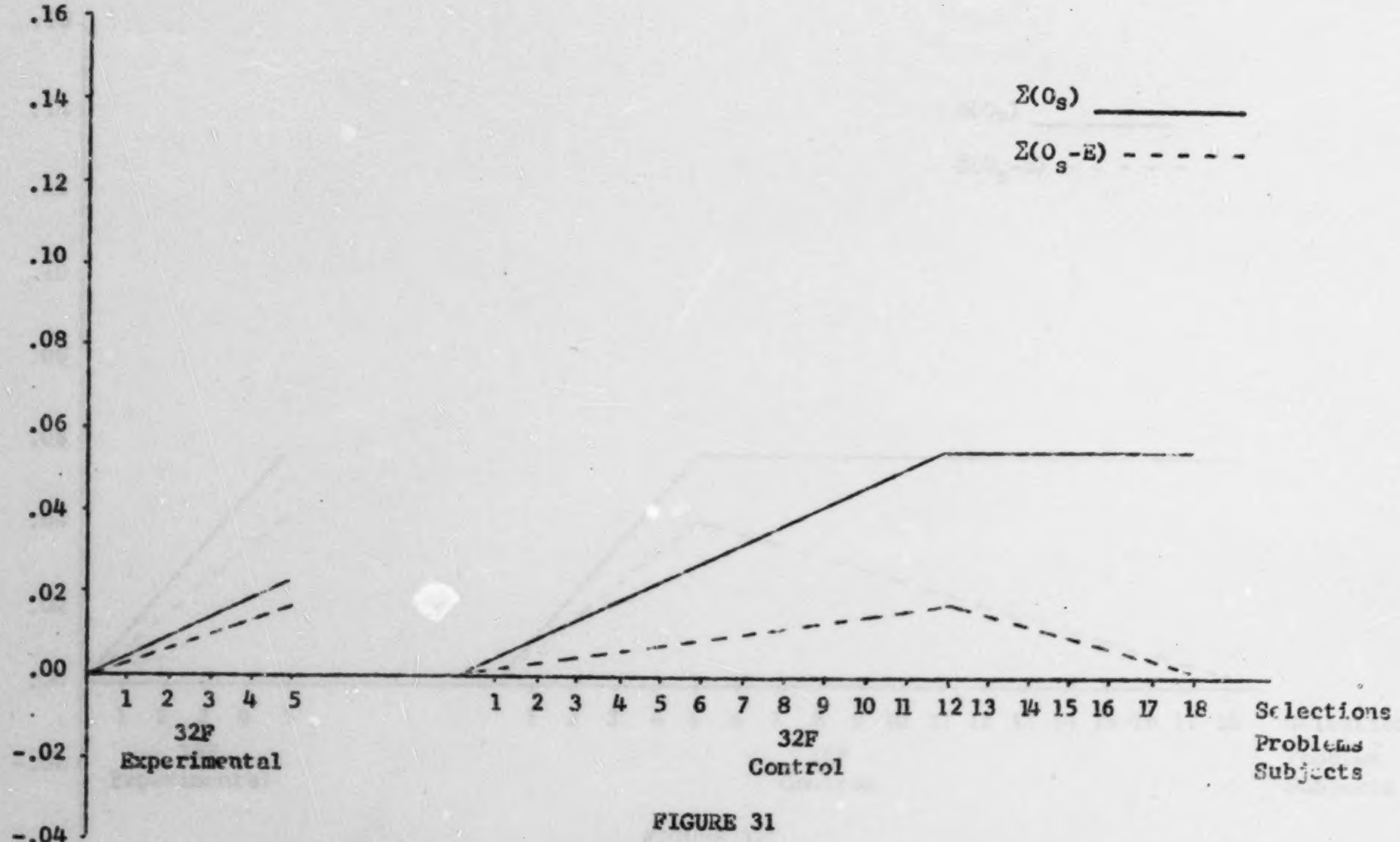

FIGURE 31

PERFORMANCE CURVES FOR A CONTROL-EXPERIMENTAL PAIR (SCHEMATA NORMS)
FOR PROBLEM 32F POST-TESTING SESSIONS.

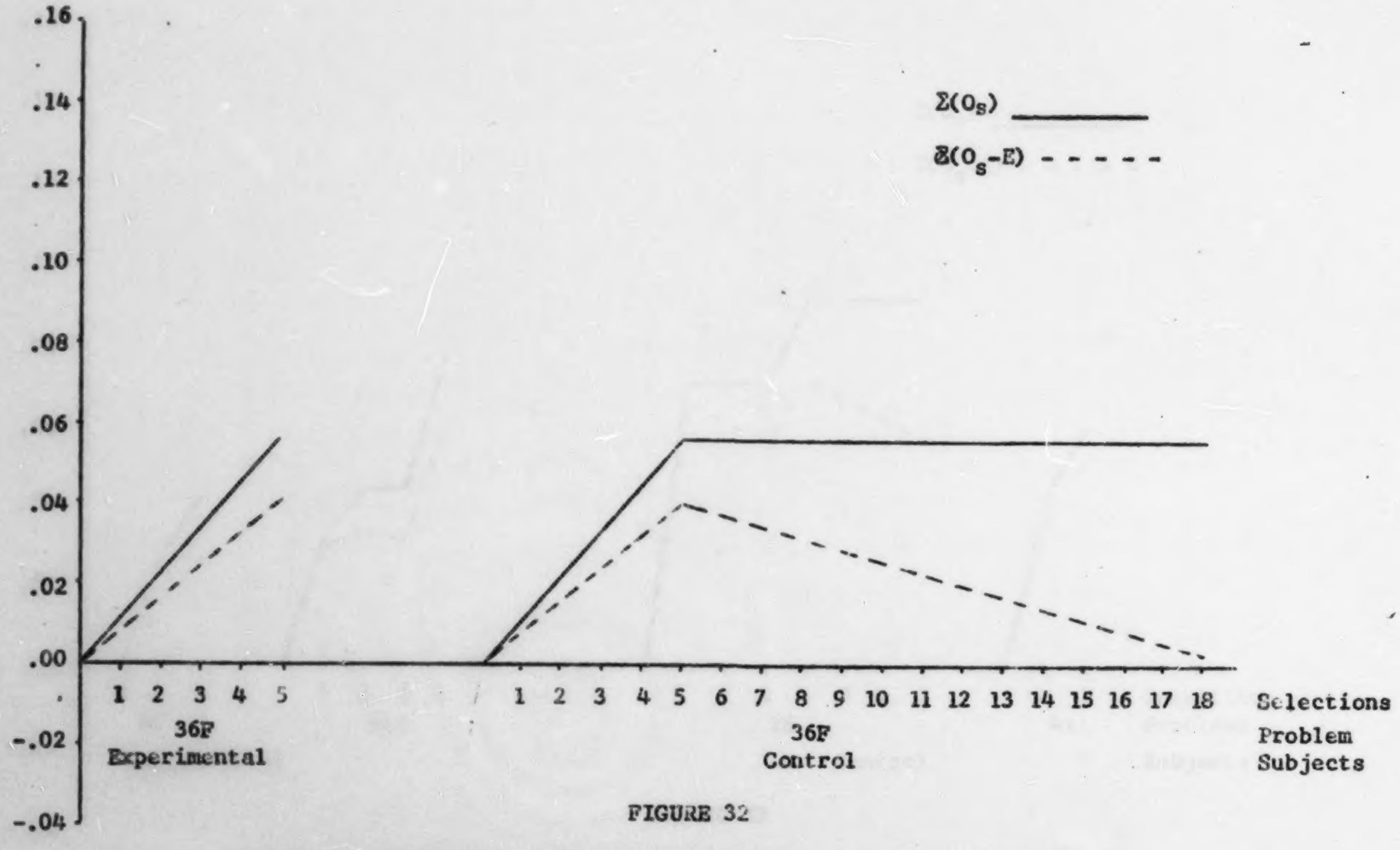

FIGURE 32

PERFORMANCE CURVES FOR A CONTROL-EXPERIMENTAL PAIR (SCHEMATA NORMS)
FOR PROBLEM 36F POST-TESTING SESSIONS

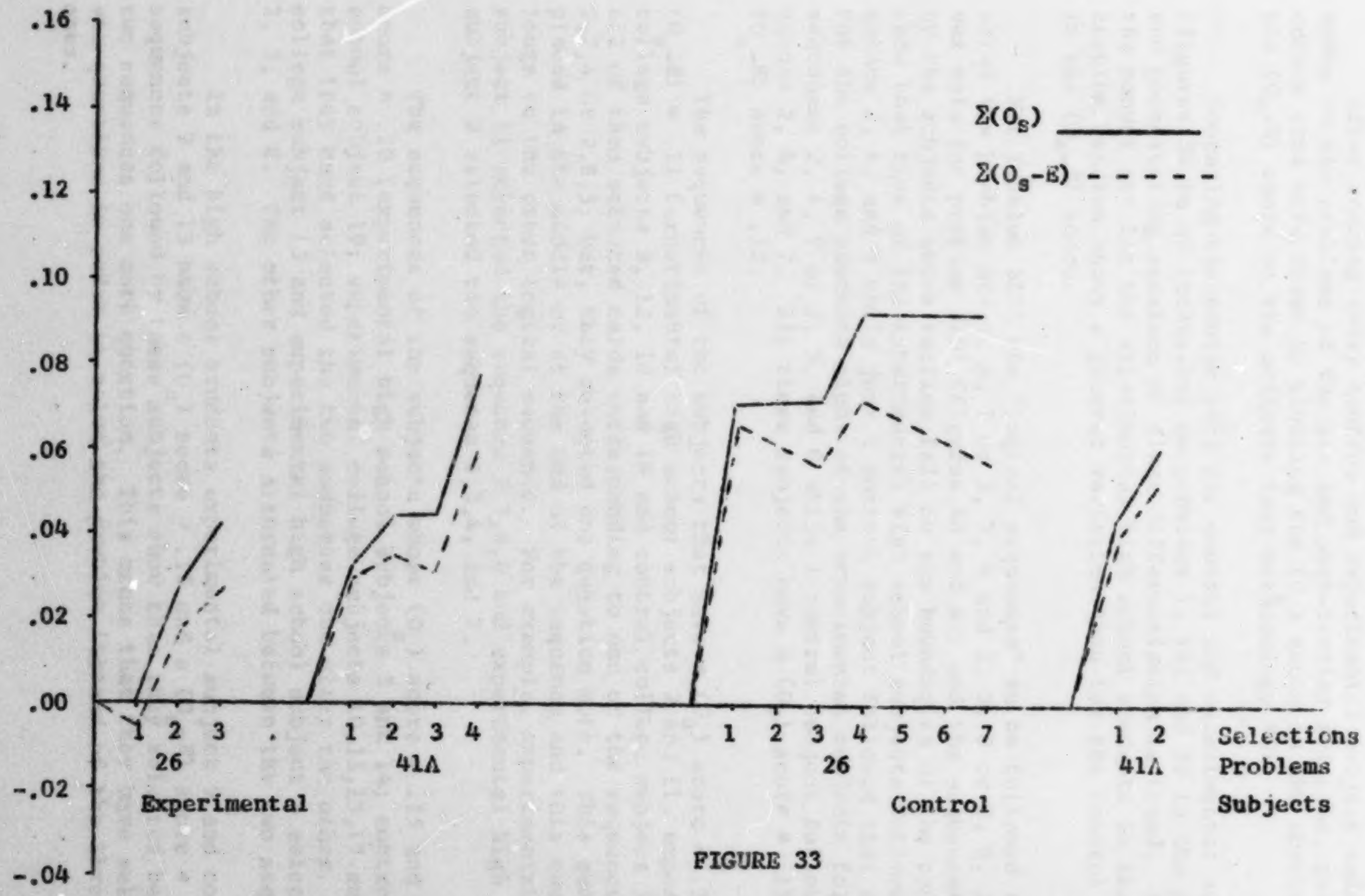

FIGURE 33

PERFORMANCE CURVES FOR A CONTROL-EXPERIMENTAL PAIR (SCHEMATA NORMS) FOR PROBLEMS 26 and 41A POST-TESTING SESSIONS.

d) Convex sets:

After scoring every control and experimental subject using schemata norms on the problems of the pre and post-testing sessions, poligonal convex sets were drawn by plotting the (O_s) score on the abscissa and the (O_s-E) score on the ordinate (see methodology #3).

Comparing the convex sets for control and experimental subjects (figures 34 to 45 inclusive) on problems 1, 19, and 25 in the pre-testing and post-testing sessions no clear differentiation is found. In problem 1 the convex set for the experimental high school students in the post-testing session shows a greater variation than for the control subjects in the (O_s-E) score.

For problem 31D' the "logical sequences" to be followed in order to solve the problem are 2, 4, 7 or 2, 7, 4 and 2, 3, 8 or 2, 8, 3. The convex sets for problem 31D' (figures 46 and 47) and the sequences follow-by the subjects whose tactics fall on the boundaries of the convex sets show that five of the experimental high school subjects followed the sequence 2, 4, and 7 while just 1 control subject followed that sequence. For the college students eight of the experimental subjects followed the sequences 2, 4, 7 or 2, 3, and 8 while 1 control subject followed the sequence 2, 4, and 7. All these subjects have a (O_s) score = .15 and a (O_s-E) score = .12.

The sequences of the subjects that have a (O_s) score = .15 but a (O_s-E) = .11 (experimental high school subjects 2 and 11, experimental college subjects 5, 12, 16 and 18 and control college subject 18) show that all of them selected cards corresponding to one of the sequences 2, 4, 7; 2,7,4 or 2,8,3; but, they selected one question more. This question was placed in the middle or at the end of the sequence and this question belongs to the other logical sequence. For example, experimental high school subject 11 selected the sequence 2,7,4,8 and experimental high school subject 2 selected the sequence 2,3,4, and 7.

The sequences of the subjects whose (O_s) score = .15 and (O_s-E) score = .10 (experimental high school subjects 3 and 14; control high school subject 19; experimental college subjects 10,13,15,17 and 19) show that they have selected the two sequences one after the other. Experimental college subject 13 and experimental high school subject 3 selected 2,4, 7, 3, and 8. The other subjects alternated between the two sequences.

In the high school students experimental subject 9 and control subjects 9 and 13 have a (O_s) score = .15 and a (O_s-E) score = .09. The sequences followed by these subjects show that they selected beside the two sequences one more question. This means that they have selected six questions in order to solve the problem instead of the three required ones.

The subjects with a (O_s) score = .15 and a (O_s-E) score = .08 (control college subject 15 selected 7 questions; the two required logical sequences and two more questions. Experimental high school subject 4 with a (O_s) score = .15 and a (O_s-E) score = .064 had selected the sequence 2,4,3,7,9,8,1,10, and 6. It can be seen that sequence 2,4, and 7 is located among the first 4 questions he had selected).

In figure 46 the convex sets for high school subjects show that experimental subject 12 and control subject 16 have the same (O_s) score = .125. Their sequences are 2,4 (subject 12) and 2,3,6,7,8,5,4,1,9,10 (subject 16). Notice that the first two questions belong to one of the logical sequences, and, that is the reason that they have the same (O_s) score. But, while subject 12 asked no more questions and his (O_s-E) score = .10, subject 16 asked 8 more questions. He is punished for all these useless questions he has asked and his (O_s-E) score = .025.

Looking at the same figure 46 control high school subject 6 has a (O_s) score = .05 and a (O_s-E) score =.02. His sequence is 8,4,7; he selected three questions but he did not ask question number 2 which is the most important and should, according to the schemata norms, be always asked in the first place. Experimental high school subjects 7 and 18 selected the sequences 6,8,2,4,3 and 3,2,4,8,7 respectively. The only question that has a score is question number 8 for subject 7 and question number 4 for subject 18. They had selected the other required questions of the logical sequences, but in a wrong order and consequently they received a score of zero for them. The sequences of subjects 7 and 11 of the control group are 7,2,8,4,3,1 and 4,7,2,3,6,5 respectively. Their situation is similar to experimental subjects 7 and 18 so they received a score for only one question (8 and 7 respectively). The (O_s) score for the 4 subjects is .025. Nevertheless they differ in terms of the (O_s-E) score by the fact that experimental subjects 7 and 18 selected 4 questions with scores of zero, and their (O_s-E) score = .025 while control subjects 7 and 11 have 5 questions with scores of zero and their (O_s-E) score = -.035.

Control subjects 4,5, and 18 have a (O_s) score = .00. This means that they have not selected any required question in the right order. Subject 4 selected four questions and his (O_s-E) score = -.04 while subjects 5 and 18 selected 5 questions and consequently their (O_s-E) score = -.045.

A similar approach can be followed in order to complete the study of the convex set for the college students (figure 47). Control college subject 12 has a (O_s) score = .10 and a (O_s-E) score = .065. The sequence that he followed is 2,5,6,7,4,3,9,8,10. He received a score for question number 2 and a zero score for all the other questions. He selected questions 7 and 4 but in the fourth and fifth order instead of the second and third order.

Control subjects 2,5, and 13 have the same (O_s) score =.05. This means that they received a score for the second and third questions they asked. The differences in the (O_s-E) scores are due to the fact that subject 13 selected 8 questions for which he received a score of zero while subjects 2 and 5 selected only one question with a zero value. Looking at the bottom of the convex set there are 5 control subjects and 1 experimental subject that have an (O_s) score = .025. They received a score for just one question. The differences in (O_s-E) scores are due to the number of questions that they have selected with a score of zero.

In summary, it can be concluded that the convex sets for problem 31D' shows a clear differentiation between control and experimental subjects. Seventeen experimental college subjects are located in the upper boundaries of the convex set while only three control college subjects are in that place. For the high school students the ratio is 11 experimental to four control subjects.

The convex set for problem 35B' (figures 48 and 49) show that 9 of the college students followed a logical sequence while 1 of the control subjects followed a logical sequence. For the high school students 7 experimental subjects had followed one of the logical sequences while just 1 control subject did so. If one wished to do a detailed study for problem 35B', a similar approach as the one followed for problem 31D' should be performed.

Figures 50 and 51 present the convex sets for problem 32F. In figures 52 and 53 the convex sets for problem 36F are presented for the high school and college students repectively.

Notice that in these problems (type b) the sequences followed by the subjects are not presented, but the number of questions that the subject used in order to solve the problem is presented. Inspection of the figures 50 and 51 show that the convex sets for the control subjects in both high school and college students coincide on the lower boundary with the convex sets for the experimental subjects, nevertheless, the convex sets for the experimental subjects show higher values on the left upper corner.

In figures 52 and 53 the convex sets for the control subjects become a line that is located in the lower boundary of the convex set for the experimental subjects. It can be seen that only the experimental subjects asked the required or less than the required number of questions. Every control subject asked more than the required number of questions in order to solve the problem. For the high school control subjects the number of questions ranked from 6 (subjects 3 and 7) to 18 questions (subjects 5,9,11,12,13,17), that is, the maximum number of questions they can ask. For the college control students the rank goes from 9 questions (subjects 2,3,14) to 18 questions (subjects 4,8,10,15,17,19). In the experimental high school students 4 of them solved the problem with the required 5 questions, 4 subjects solved it with 4 questions and 1 subject solved it with 3 questions. In the college students 8 of them used the required number of questions in order to solve the problem. 2 subjects solved it with 4 questions and 2 other subjects solved it with 2 questions.

It can be seen that several experimental subjects solved the problem with less than the required number of questions. The problem can be solved with 4 questions if the subject assumes that the code is based on the color and kind of lines, and he starts asking questions on this basis. To follow this approach is a very "good guess" for this problem and it was followed by several experimental subjects who had similar problems in the training sessions, but, computing the schemata norms on 4 questions will punish the control subjects who do not know the problem at all and had no "good guessing" approach. It was decided that with 5 questions the problem could be solved even if the subject had no idea of the rationale underlying the assignment of values to the different areas. The convex sets for problem 36F show a clear differentiation between control and experimental subjects.

The convex sets for problem 26 in the high school students (figure 54) show higher values for the experimental than for the control subjects. In the convex set for college students (figure 55) the experimental subjects show lower scores than the control subjects.

The convex set for problem 41A in the high school students (figure 56) shows a greater variation, in terms of the (O_s) score, for the experimental subjects than for the control subjects; the reverse is observed on the (O_s-E) score. In the convex set for the college students (figure 57) the higher values are observed for the experimental subjects.

A detailed study of the convex sets was presented here for problems 31D' and 36F. These two problems have been selected because their schemata is very clear and a small number of questions are required in order to solve the problems. There is no other reason and a similar study can be performed with any one of the convex sets for any problem.

In summary, the study of the convex sets permit one to differentiate between the control and experimental subjects. It is possible to see the sequence or tactic followed by each subject in order to solve the problem. When the convex sets are based on the schemata norms as in the cases described here, it is possible to see the subject that has followed a "logical" sequence, he will be at the upper corner of the convex set. It is also possible to see how a subject departed from the "logical" sequences. It seems that when the subject starts asking the questions in a "logical" manner but does not finish the sequence, which means that he has asked less than the required number of questions, his tactic will be located on the upper horizontal boundaries of the convex set. The fewer the questions he asked the lower will be his position on that boundary. The subject who falls on the lower corner of the convex sets will be the subject who has asked none or very few of the questions that belongs to any "logical" sequence. His location on that boundary will depend on the number of useless questions he has asked. The subject who starts asking questions in a "logical" sequence but then departs from that sequence and asked questions at random will be located on the lower horizontal boundaries of the convex sets. Again, his position on that boundary will depend on the number of questions he has asked in a "logical" way. The subjects who asked all the questions that belong to one of the "logical" sequences according to schemata norms, will be located on the upper vertical boundary of the convex set. If he asked just the required questions, he will fall in the upper corner of the boundary. But, if beside the required questions he has asked others, his location will fall lower on that upper vertical boundary according to the number of useless questions asked.

The subjects whose tactics are located inside the convex sets are the ones who asked as many questions of a "logical" sequence as the subjects who are located on the same ordinate on the boundaries. Their position along that ordinate will depend on the number of useless questions asked.

All these implications can be seen by a close inspection of figures 46,47,48,49,52,53,56 and 57 that present the convex sets for problems 31D', 35B',36F and 41A which are the problems with a clear schemata.

There are cases where the problem can be solved using different sequences or tactics, but not all of them have the same weights. In other words, if there are several "logical" tactics, there is one that is "more logical" than the others or there is a group of questions that should be asked. There is no absolute position in the sequence for every question. The results are that several of the "logical" sequences will have different scores and the subjects who followed them will be located at different points on the convex sets. This is the case of experimental college subjects 3,12,and 17 and control college subjects 4 and 1 on problem 26 (figure 55) who have followed one of the "logical" sequences, nevertheless, their locations are different on the convex sets.

Looking at the convex set for the same problem (26) in high school students (figure 54), it can be seen that experimental subject 11 had followed a "logical" sequence. However, his score is lower than subject 19 who had selected a "logical sequence" but with two more questions at the end. This happened because according to the schemata norms subject 19 had selected a sequence with more weights than the sequence selected by subject 11.

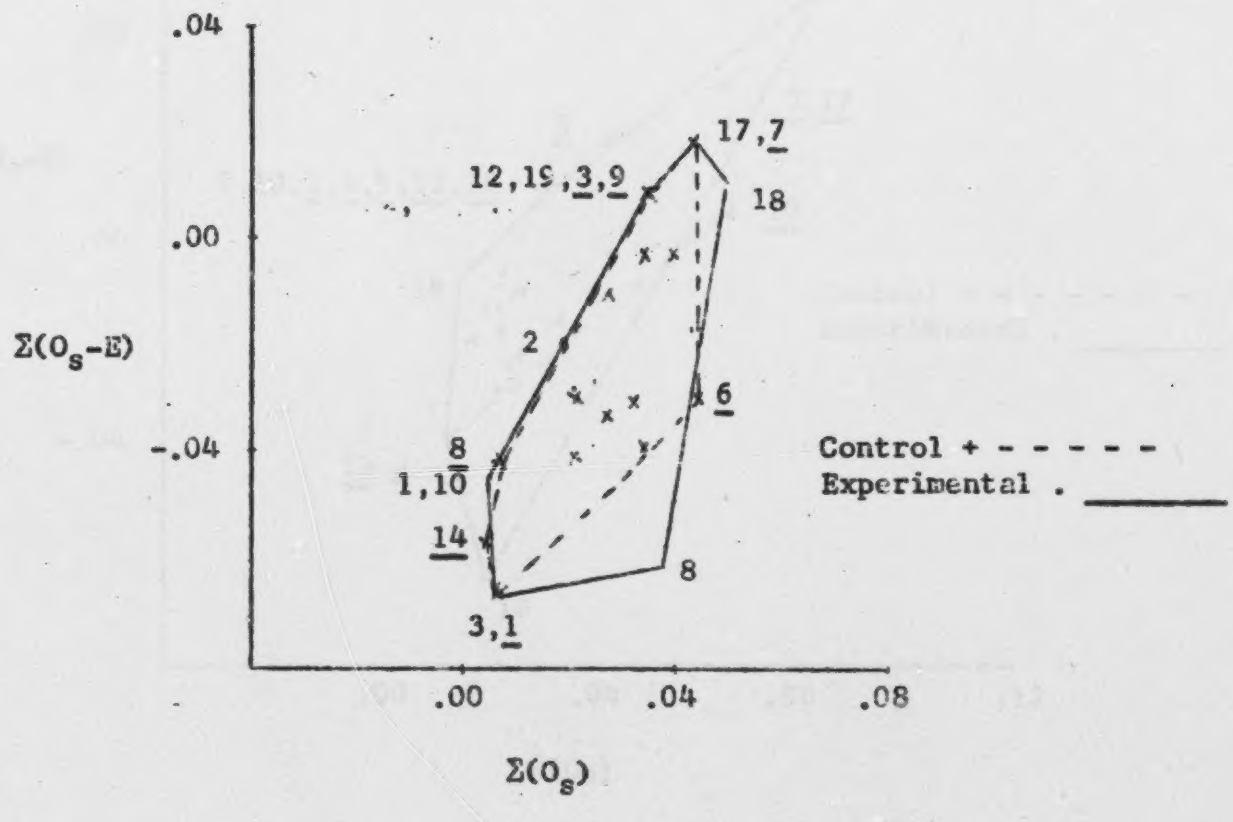

FIGURE 34

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 1 OF THE PRE-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control

Subjects	Tactics
7	4,7
3,9	4,8
8	1,2,4,5
14	1,2,5,9,4
1	1,2,3,5,6,7
6	3,4,8,7,9,6

Experimental

Subjects	Tactics
17	3,7
12	4,5
19	4,8
2	1,5,3
1	2,1,8,3
10	1,2,8,4
3	2,1,3,7,4,8
8	5,7,1,4,3,2,7,8
18	4,7,8

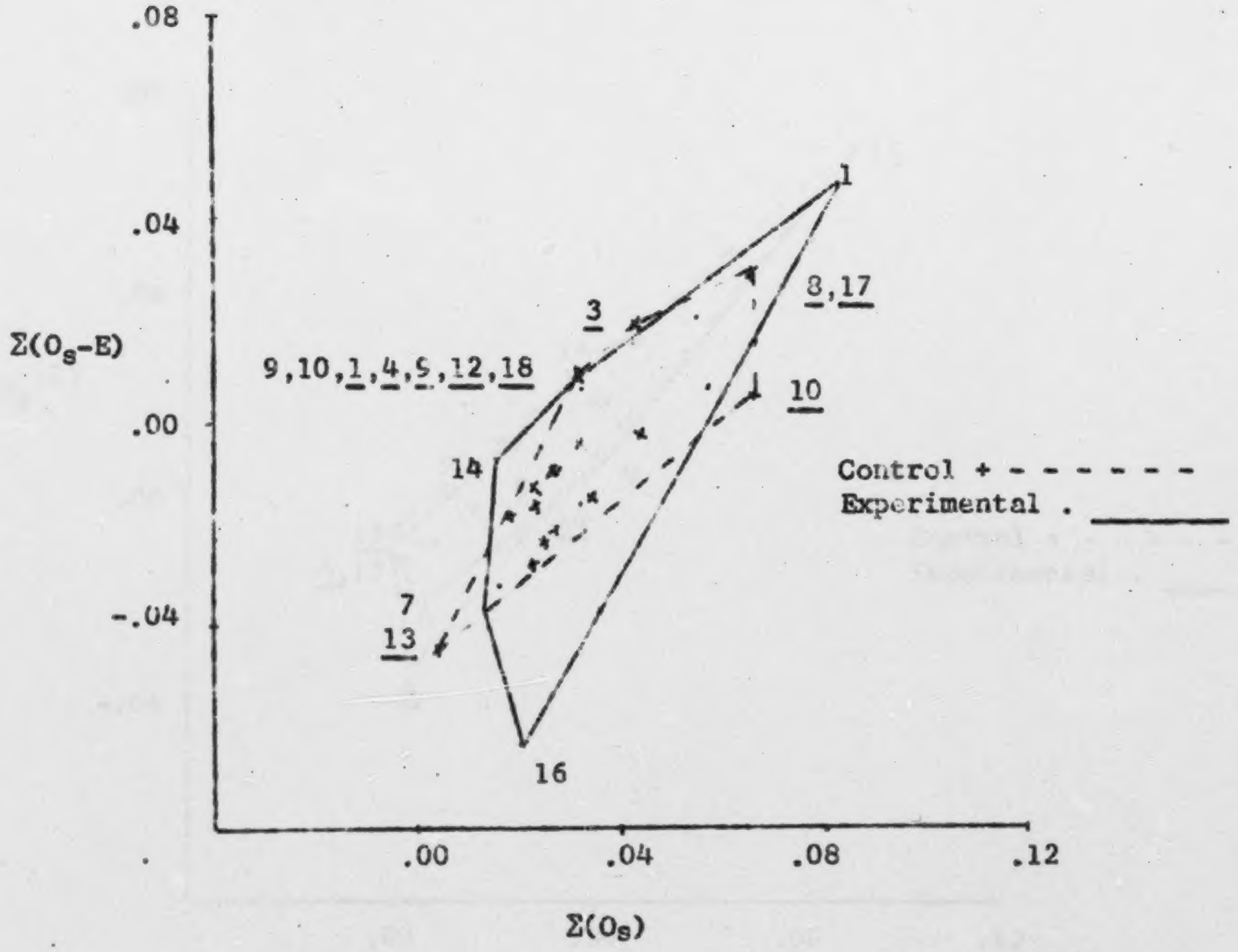

FIGURE 35

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 1 OF THE PRE-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
8	4,5,7	1	4,6,7
17	4,8,7	9	4,8
3	4,7	10	4,5
1,18	3,8	14	5,2
4,12	4,8	7	2,9,3,1
9	4,5	16	1,3,5,7,9,2,4
13	1,2,8,4		
10	4,5,7,6,3		

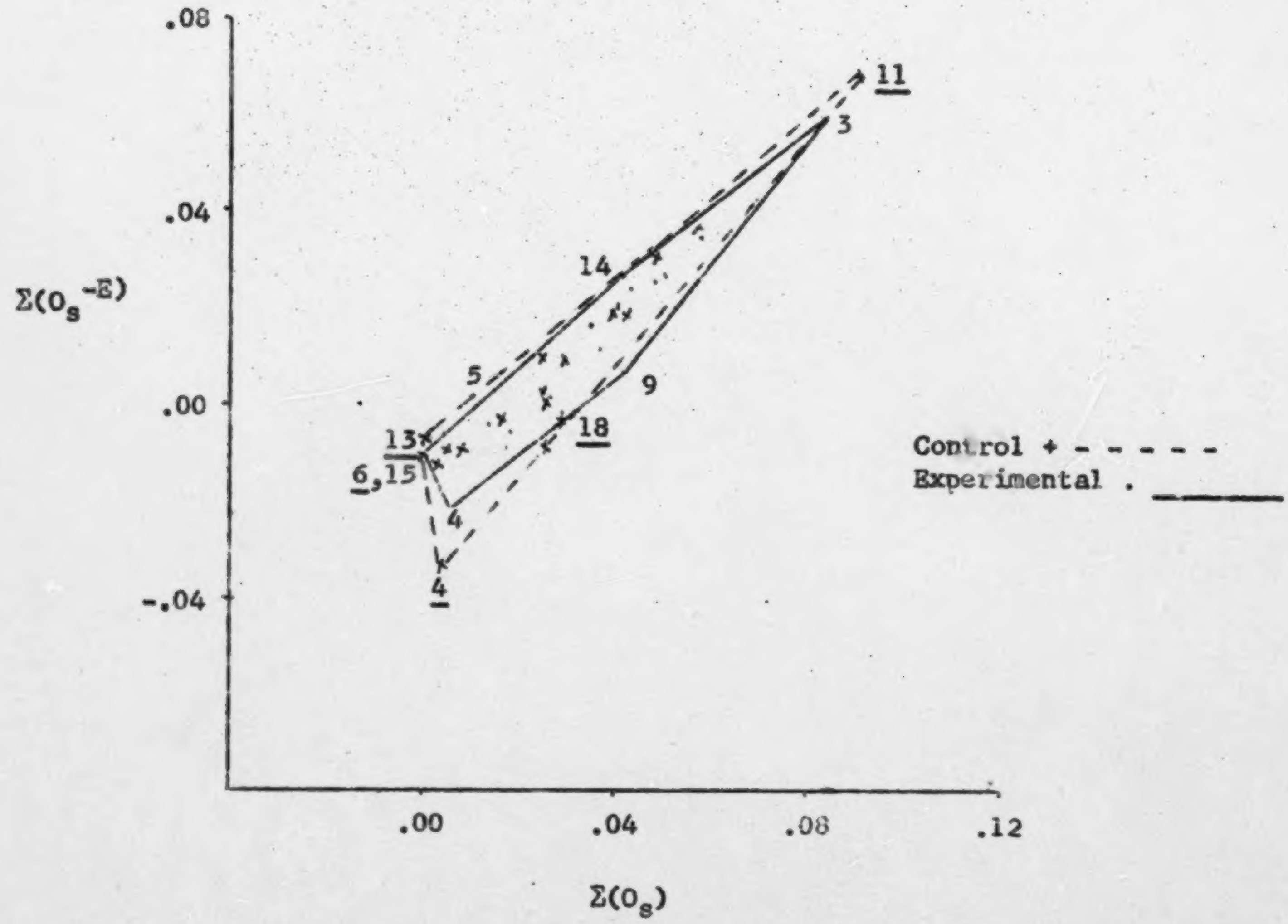

FIGURE 36

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 19 OF THE PRE-TESTING SESSIONS BASED ON SCHEMATA NORMS

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
11	6,15,16,17,2,18,4,19,5	3	6,16,17,15,2,18,4,19,5,20,8,12
13	5,12,13	14	5,8,2,6,4,18
6	5,12,14,13	5	9,15,2,18,4
4	5,9,12,13,14,2,19,18,20, 6,17,16,15,7,4	15	5,14,13,12
18	9,12,13,14,7,16,19,2,4, 5,15,18,20,17	4	5,12,6,17,15,13,9,19,20,18,4
		9	2,18,4,19,5,20,6,15,8,12,14,9, 11,1

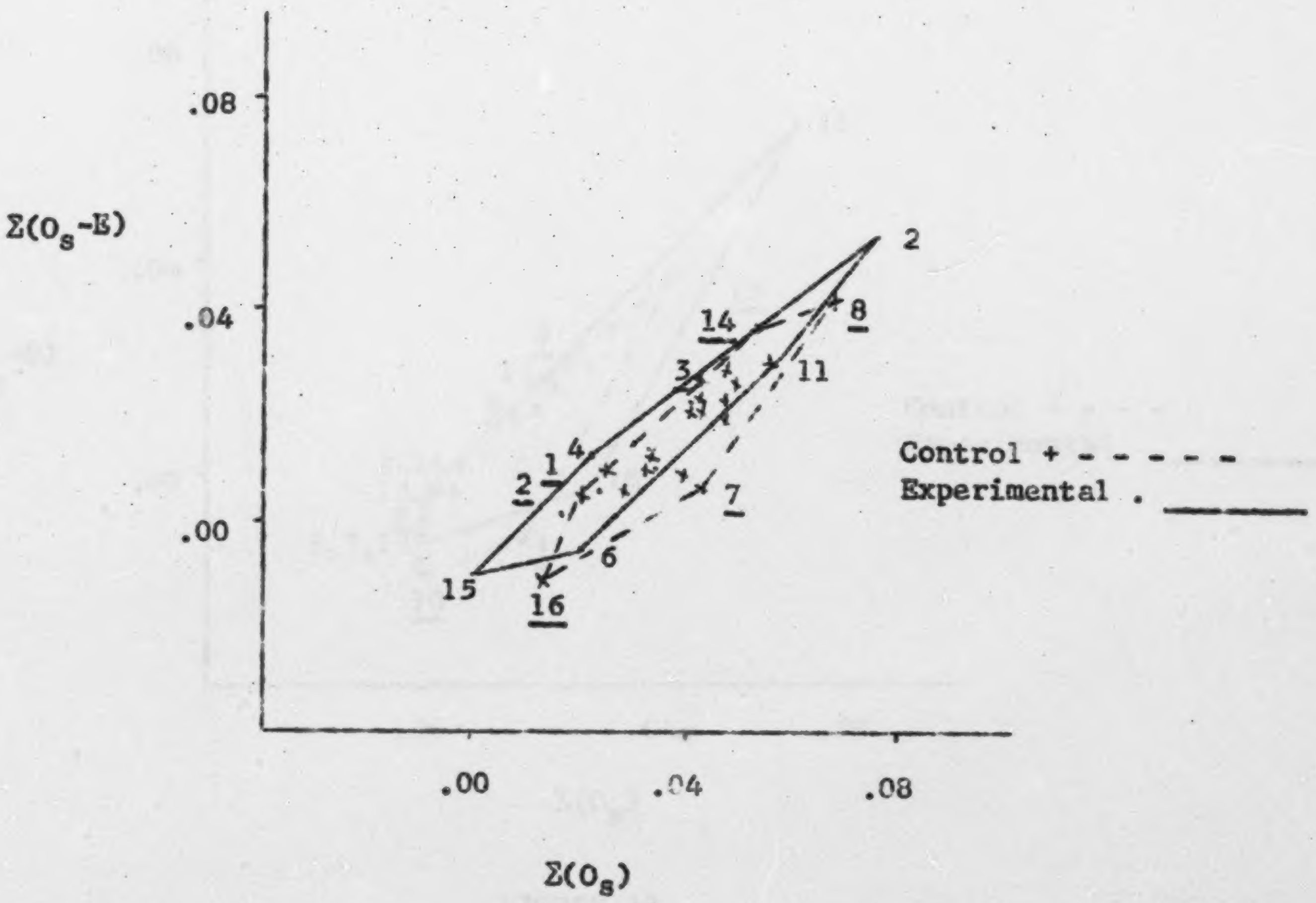

FIGURE 37

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 19 OF THE PRE-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
8	6,15,17,2,19,5	2	19,2,16,6,17,18,4,15,5
14	2,6,15,16,5,19,17	4	6,2,19,5
3	6,16,19,5,2,17	15	5,14,13,12
1	2,19,5,18,4,17	6	2,6,7,19,5,9,12,20
2	5,2,20,19,18,14	11	2,20,18,4,5,19,15,17,16,6
16	5,8,9,15,17,6,16,2,18,19		
7	9,12,13,14,2,18,19,20,4, 1,5,6,15,17		

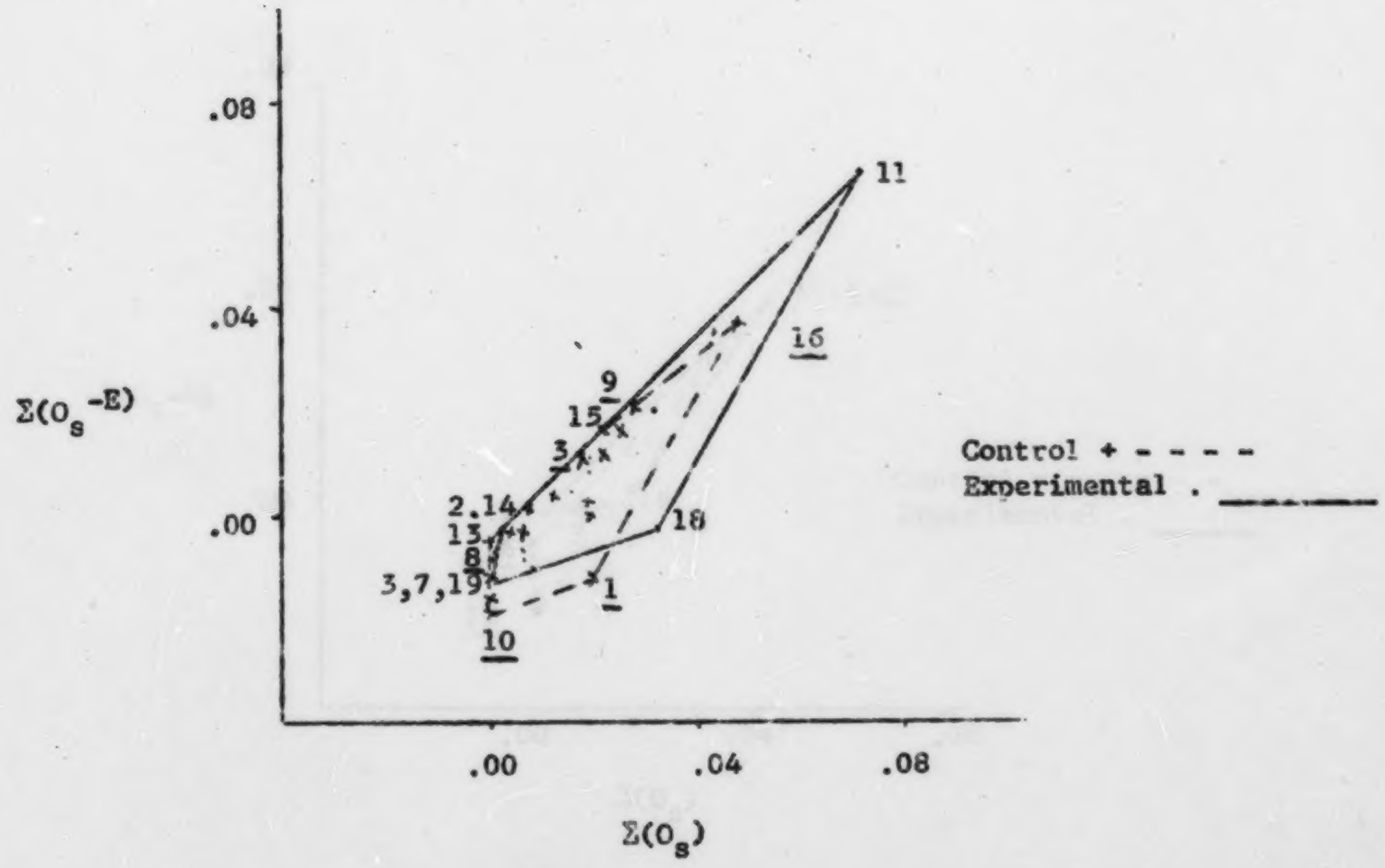

FIGURE 38

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 25 OF THE PRE-TESTING SESSIONS BASED ON SCHEMATA NORMS

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
16	9,2,11,12,15,18	11	10,2,23,24,20
9	1,2,3,10,20	15	8,7,3,9,24,20
3	9,19,7,15	2	2,7,9,20
8	1,9,24,16	14	25,23,11,15
10	1,3,2,4,8,9,12,14,15,16,17,	13	2,4,12,22,11,25
	18,24	3	23,24,25,9,14,15,16,18,26
1	1,2,3,4,5,6,7,8,9,10,12,13,	7	1,9,4,6,12,17,23,24,18
	14,15,16,17,18,19,23,24	19	12,9,1,2,4,24,15
		18	2,3,10,20

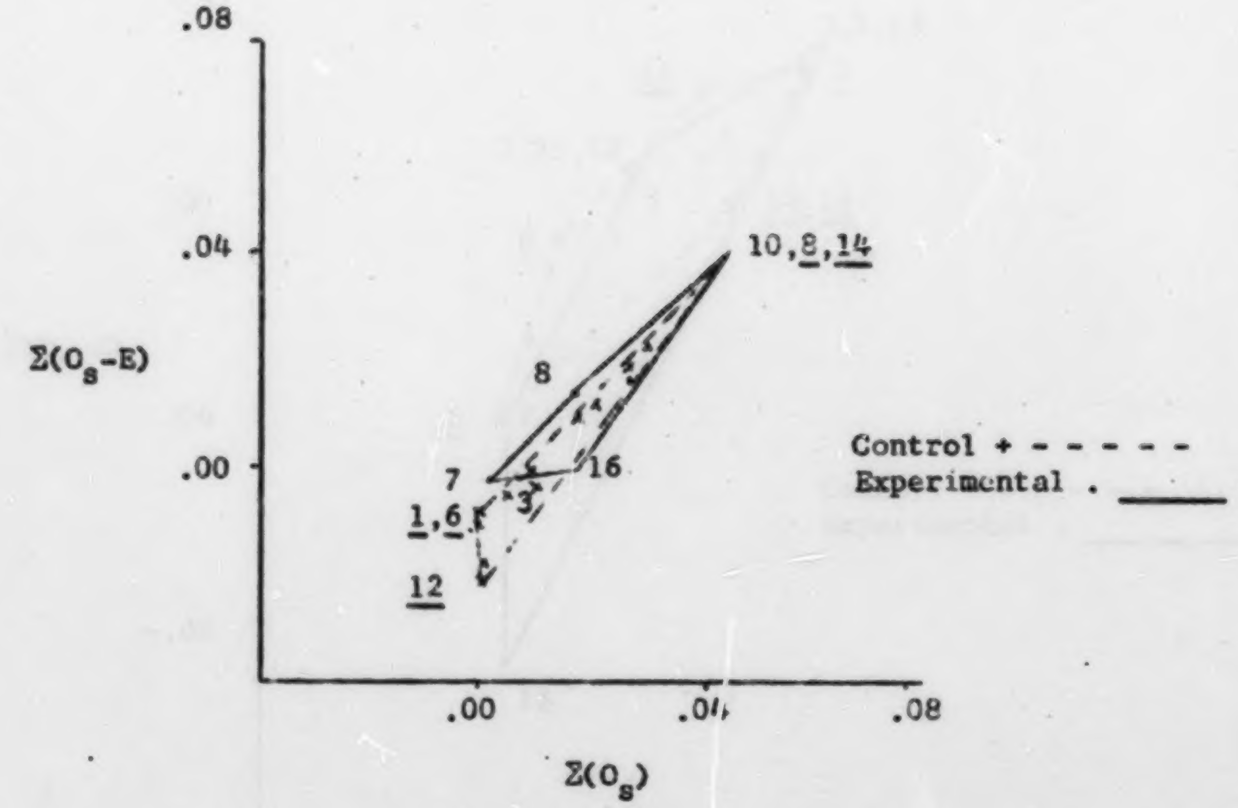

FIGURE 39

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 25 OF THE PRE-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
8	10,23,2,24,20	10	9,2,23,15,5
14	1,2,9,24,20	8	10,25,20
1	2,3,1,9,5	7	1,10,8,20
6	1,9,15,11,18	3	2,1,3,4,9,15
12	2,3,1,4,9,6,11,13,14,16,15,20,22,21	16	1,2,3,4,5,9,14,17,24,25,15,20

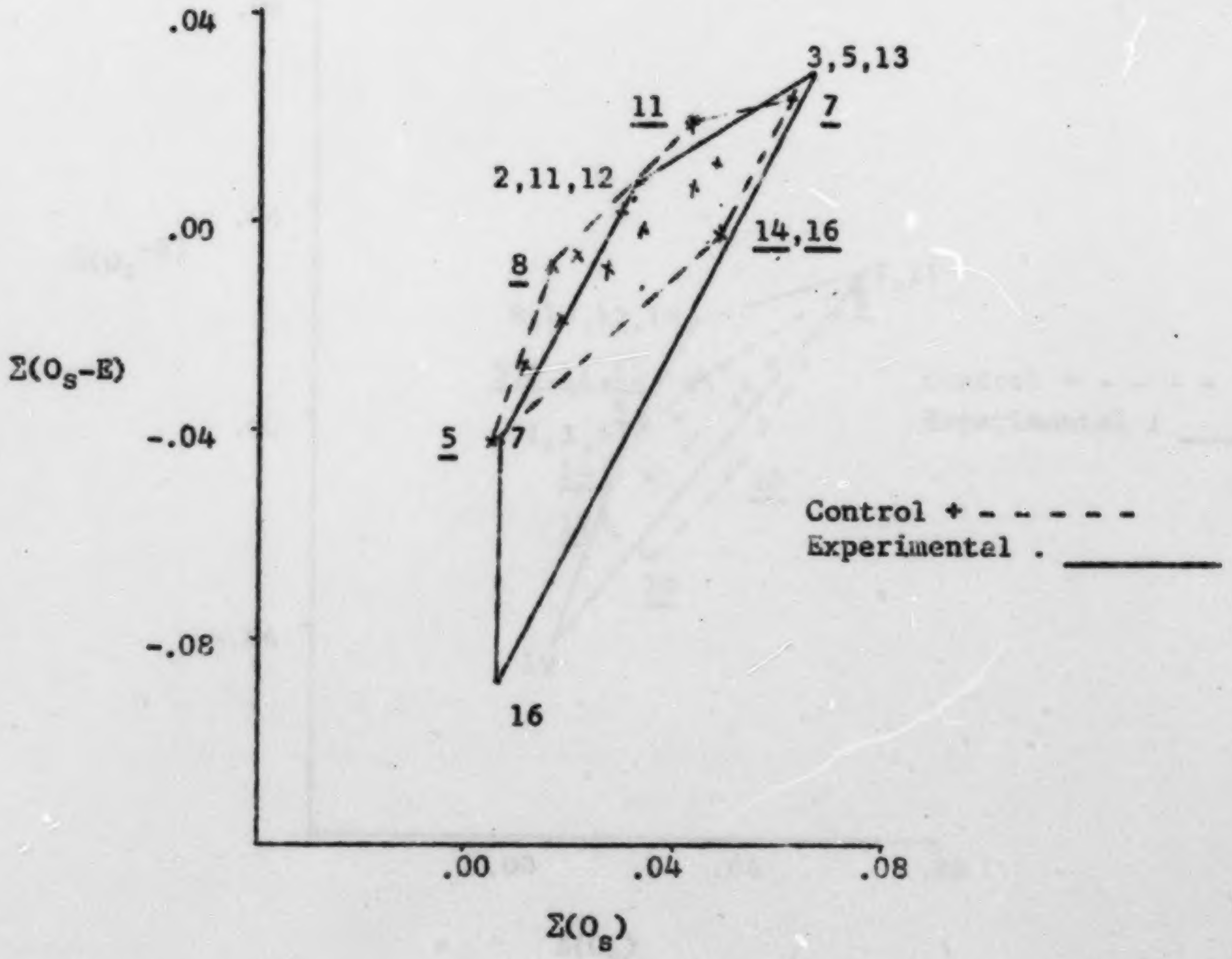

FIGURE 40

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 1 OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
7	3,9,7	3,5	4,8,7
11	4,7	13	3,8,7
8	2,3	2,12	4,8
5	2,1,9,4	11	4,7
14	3,7,8,2	7	1,2,3,5
16	9,6,8,4	16	1,2,3,4,5,9,8,7

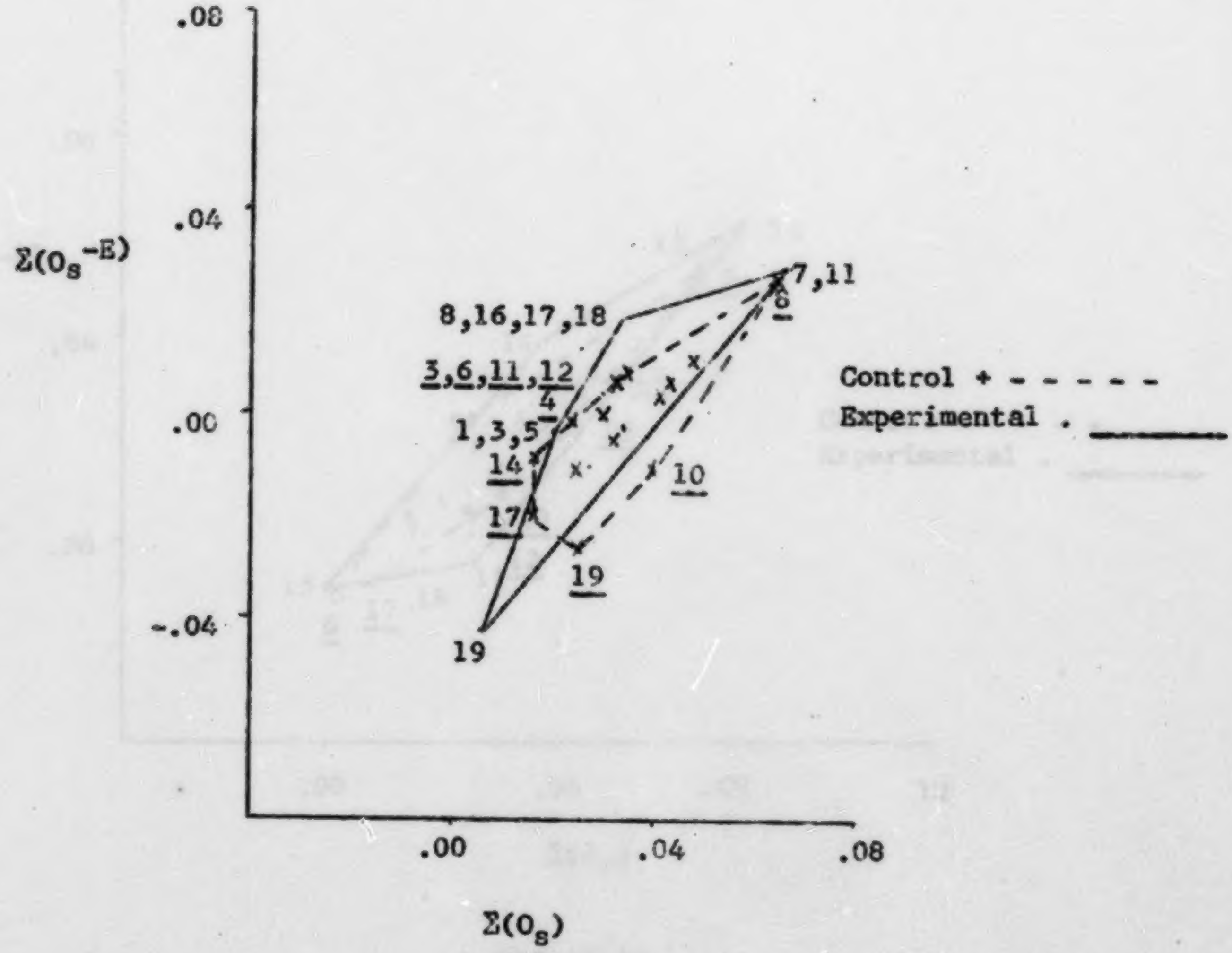

FIGURE 41

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 1 OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control

Subjects	Tactics
8	8,3,7
3,6	4,5
11	3,8
12	4,8
14	6,9
17	2,5,8
19	5,1,4,6
10	4,5,8,7

Experimental

Subjects	Tactics
7	4,5,7
11	4,8,7
8,16,17,18	4,8
1,3,5	6,8
19	2,1,4,9

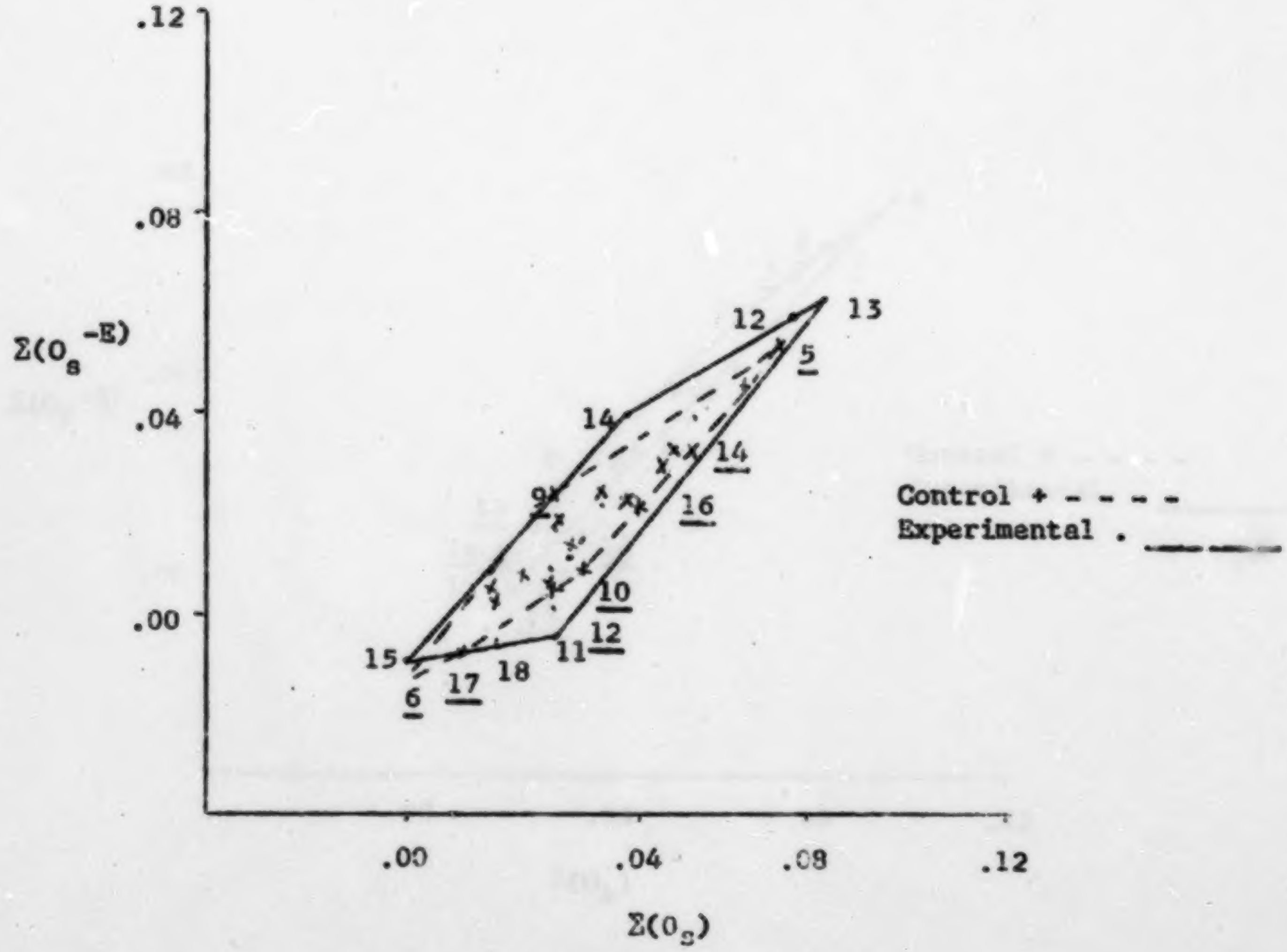

FIGURE 42

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 19 OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
5	6,16,15, 17,2,18,19,20	13	6,15,16,17,18,4,2,19,5
9	2,18,19,20,5,4	12	6,15,16,17,2,19,5
6	5,9,12,14,13	14	2,18,4,6,15,17,16,19,5
17	2,6,5,9,13,14,17,16	15	12,13,14,5
12	2,18,4,19,5,20,12,6,15,17	18	18,2,4,19,20,17,6,16,15,5
10	9,18,20,19,5,2,4,6,15,17	11	2,18,19,20,4,8,12,13,14,7,15,16,17,9
16	2,18,4,19,20,5,6,15,17,16		
14	2,20,19,18,4,5,6,8,15		

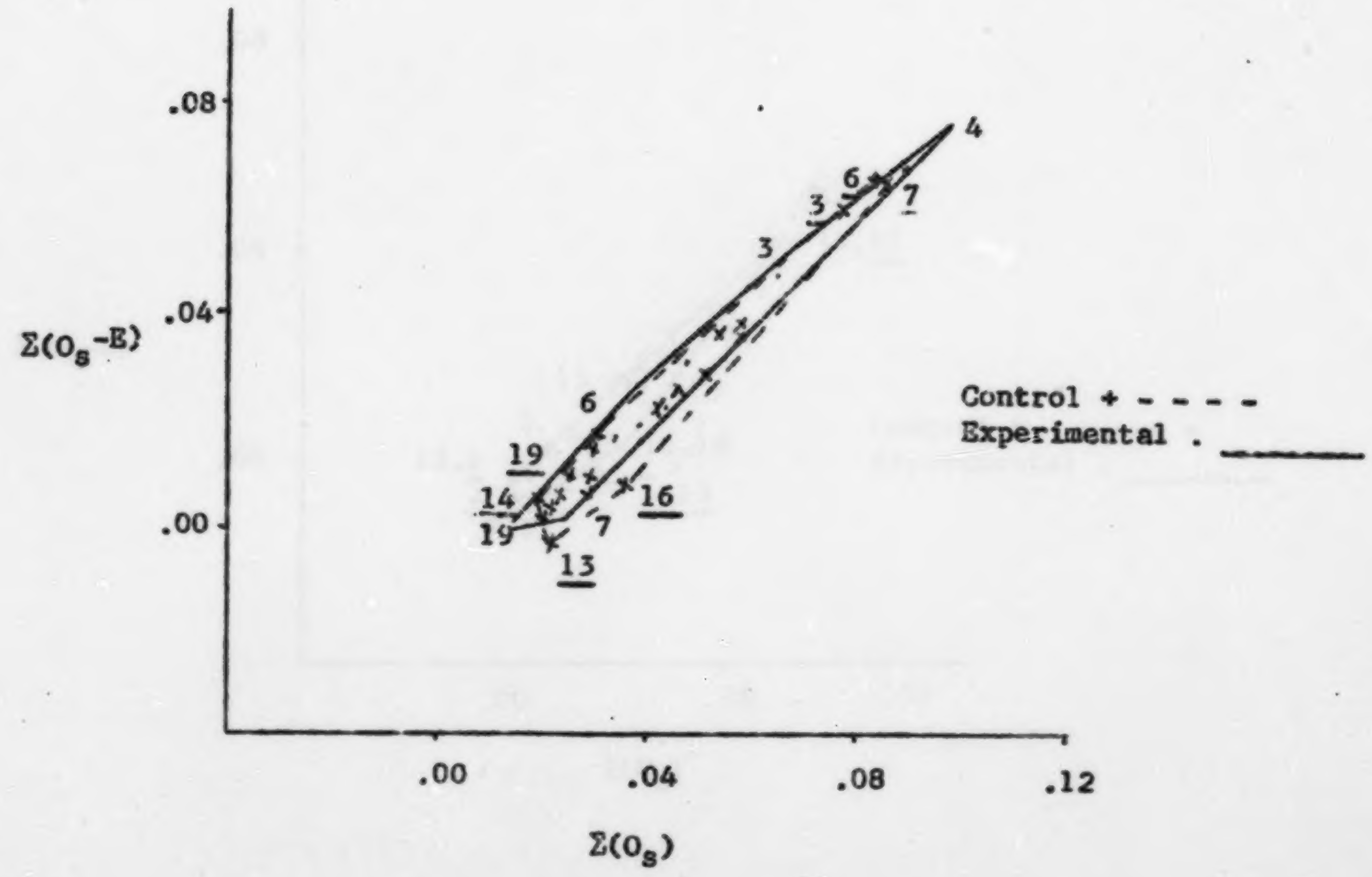

FIGURE 43

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 19 OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS

Control

Subjects	Tactics
7	6,15,16,17,2,18,19,5
6	6,15,16,2,18,19,5
3	6,16,15,17,2,19,5
19	2,20,18,19,5,17
14	2,19,7,15,17
13	2,18,4,19,20,6,7,16,17,5
16	2,20,6,15,8,1,18,4,19,5,17

Experimental

Subjects	Tactics
4	6,15,16,17,2,18,19,4,5
3	6,15,17,2,19,5
6	6,2,17,19,5
19	20,19,18,5,6,2
7	5,6,15,16,17,4,9,12,13

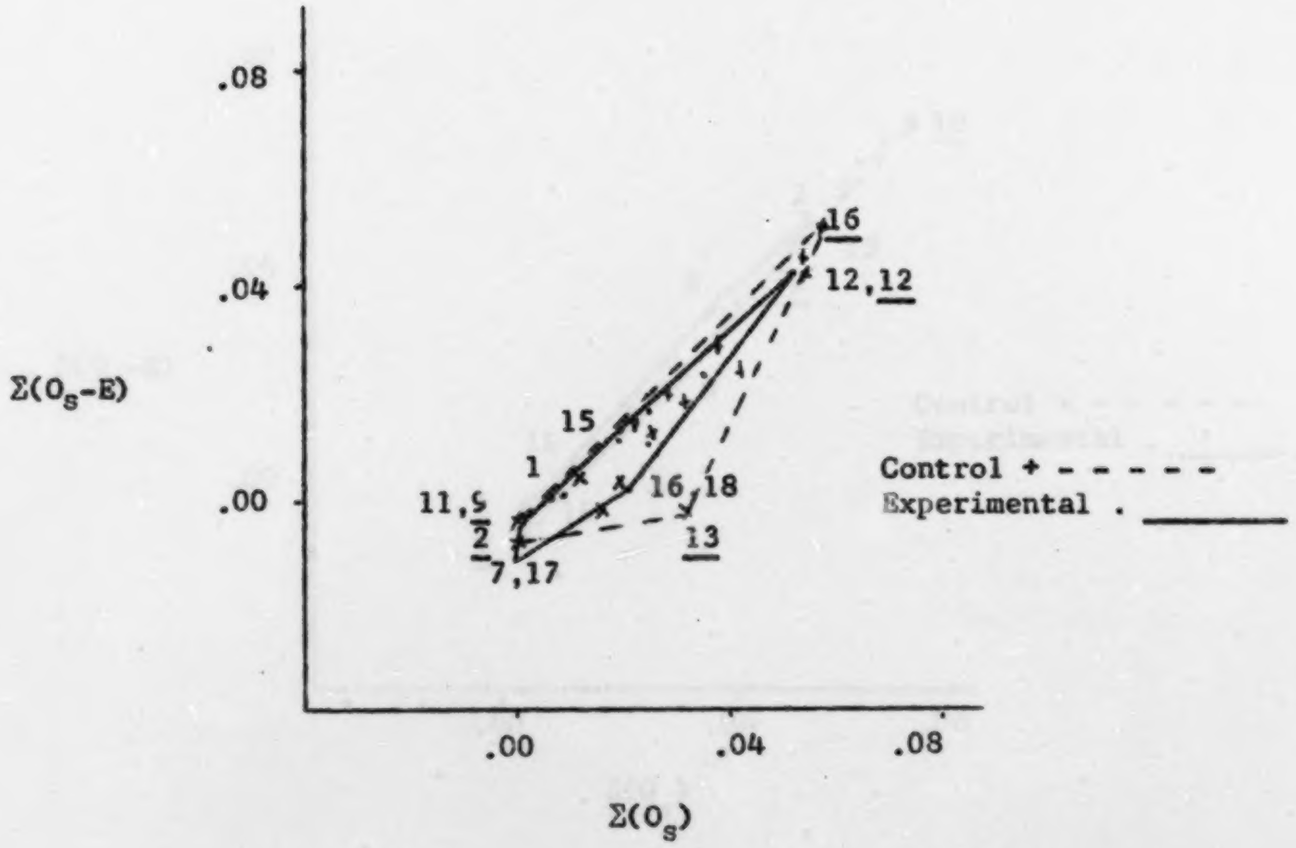

FIGURE 44

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 25 OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control

Subjects	Tactics
16	10,2,23,15,20
9	2,9,1,20
2	7,9,8,26,17,18
13	2,23,15,20
12	10,8,23,20

Experimental

Subjects	Tactics
12	9,2,1,15,20
15	1,2,10,26
1	26,25,23,17
11	2,9,11,20
7	2,3,10,7,17,16,21,24
17	5,6,7,8,11,20,14,17,23
16	1,2,3,5,6,9,10,24,23,17,4,26
18	1,8,6,9,3,18,17,24,23,21,22,16

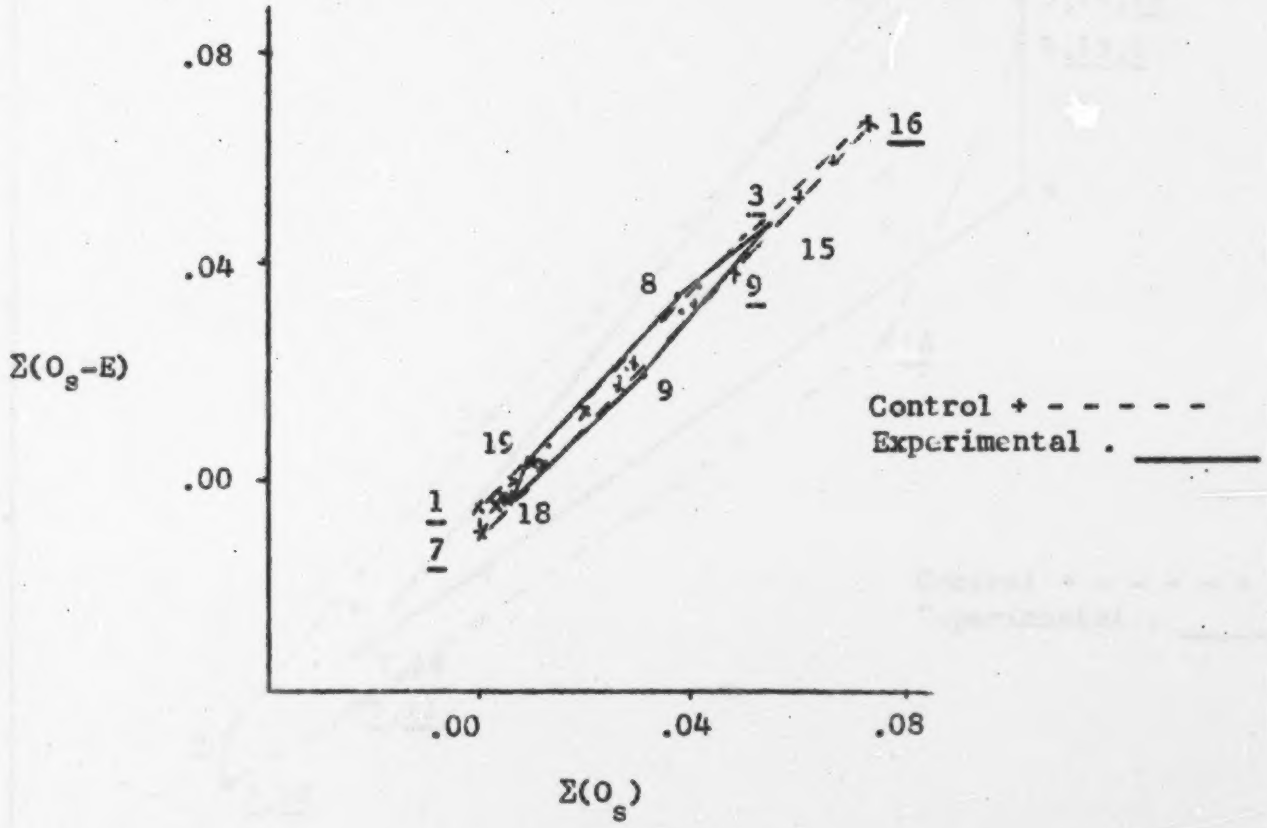

FIGURE 45

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 25 OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
16	9,2,23,24,15	15	9,8,23,26,20
3	9,15,18	8	9,8,20
1	18,24,8,4	19	2,3,7,9,20
7	1,10,23,26,18,17,15	18	2,3,5,9,24,23,15
9	9,23,2,24,20	9	7,8,3,9,14,15,24

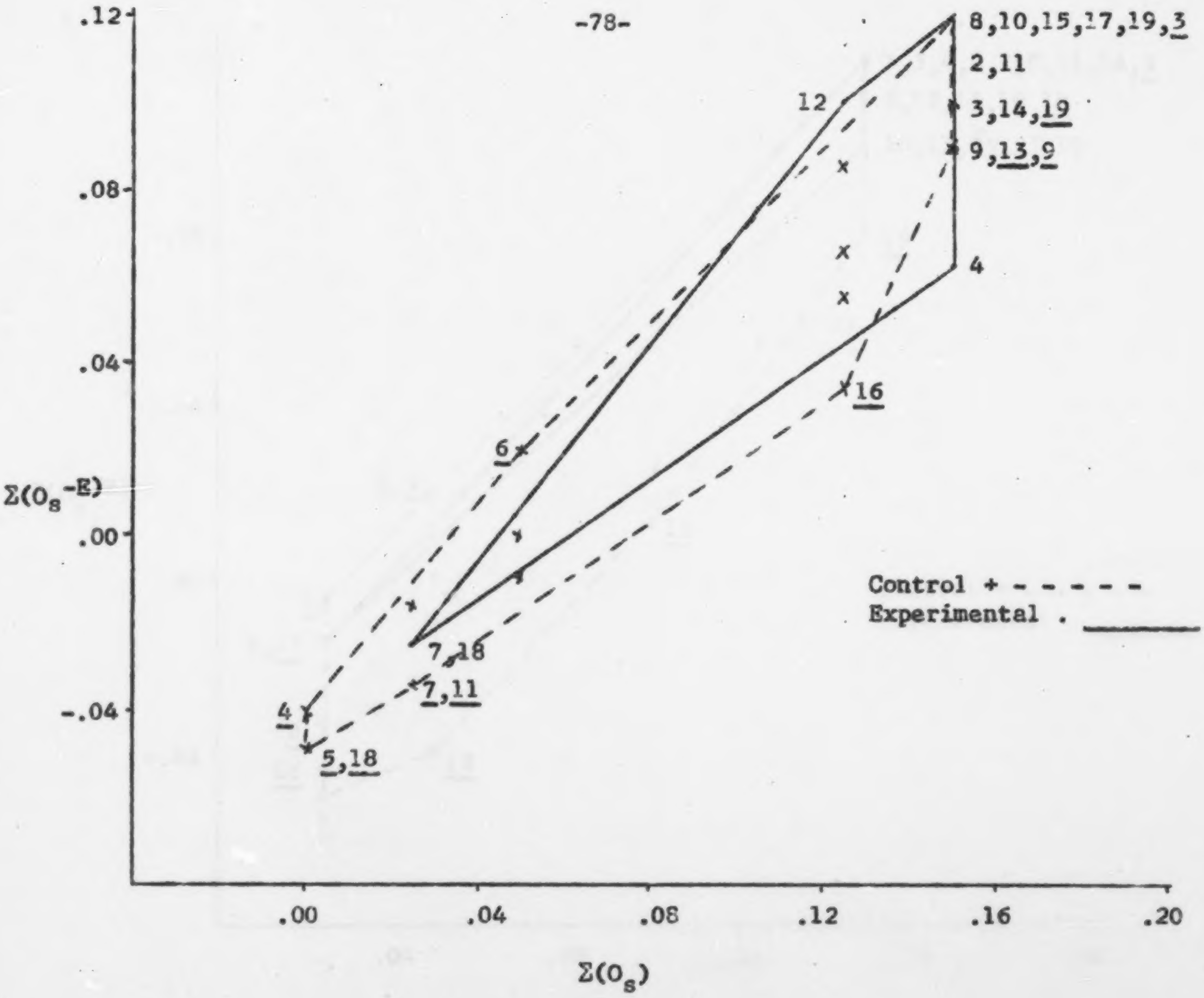

FIGURE 46

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 31D' OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
3	2,4,7	8,10,15,17,19	2,4,7
6	8,4,7	12	2,4
4	1,5,2,7	7	6,8,2,4,3
5	4,10,6,3,5	18	3,2,4,8,7
18	8,2,5,7,3	4	2,4,3,7,9,8,1,10,6
7	7,2,8,4,3,1	9	2,3,4,7,8,6
11	4,7,2,3,8,5	3	2,4,7,3,8
16	2,3,6,7,8,5,4,1,9,10	14	2,4,3,7,8
9,13	2,4,8,3,7,6	2	2,3,4,7
19	2,4,8,3,7	11	2,7,4,8

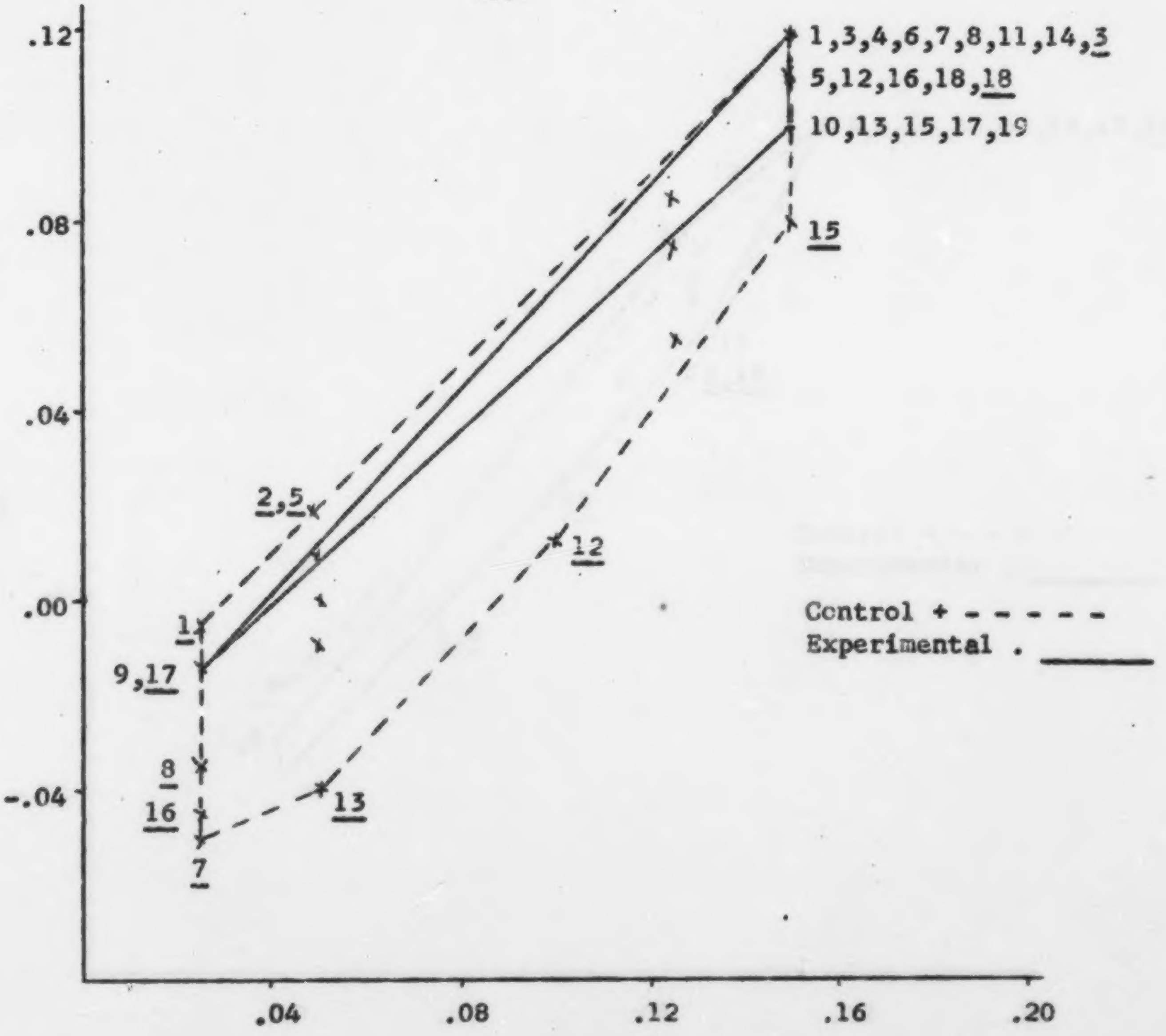

FIGURE 47

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 31D' OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
3	2,4,7	1,4,7,11,14	2,4,7
2,5	6,7,8	3,6,8	2,3,8
1	7,2,4	9	6,2,4,7
17	7,2,8,4	10	2,3,7,4,8
8	6,2,3,4,7,8	13	2,4,7,8,3
16	8,1,7,3,5,6,4	15	2,4,3,8,7
7	4,7,2,8,6,3,5,9	17	2,3,7,8,4
13	3,8,7,6,4,2,1,5,9,10	19	2,4,8,7,3
12	2,5,6,7,4,3,9,8,10	12,18	2,7,8,3
15	2,3,4,8,6,7,5	5,16	2,4,3,7
18	2,7,8,3		

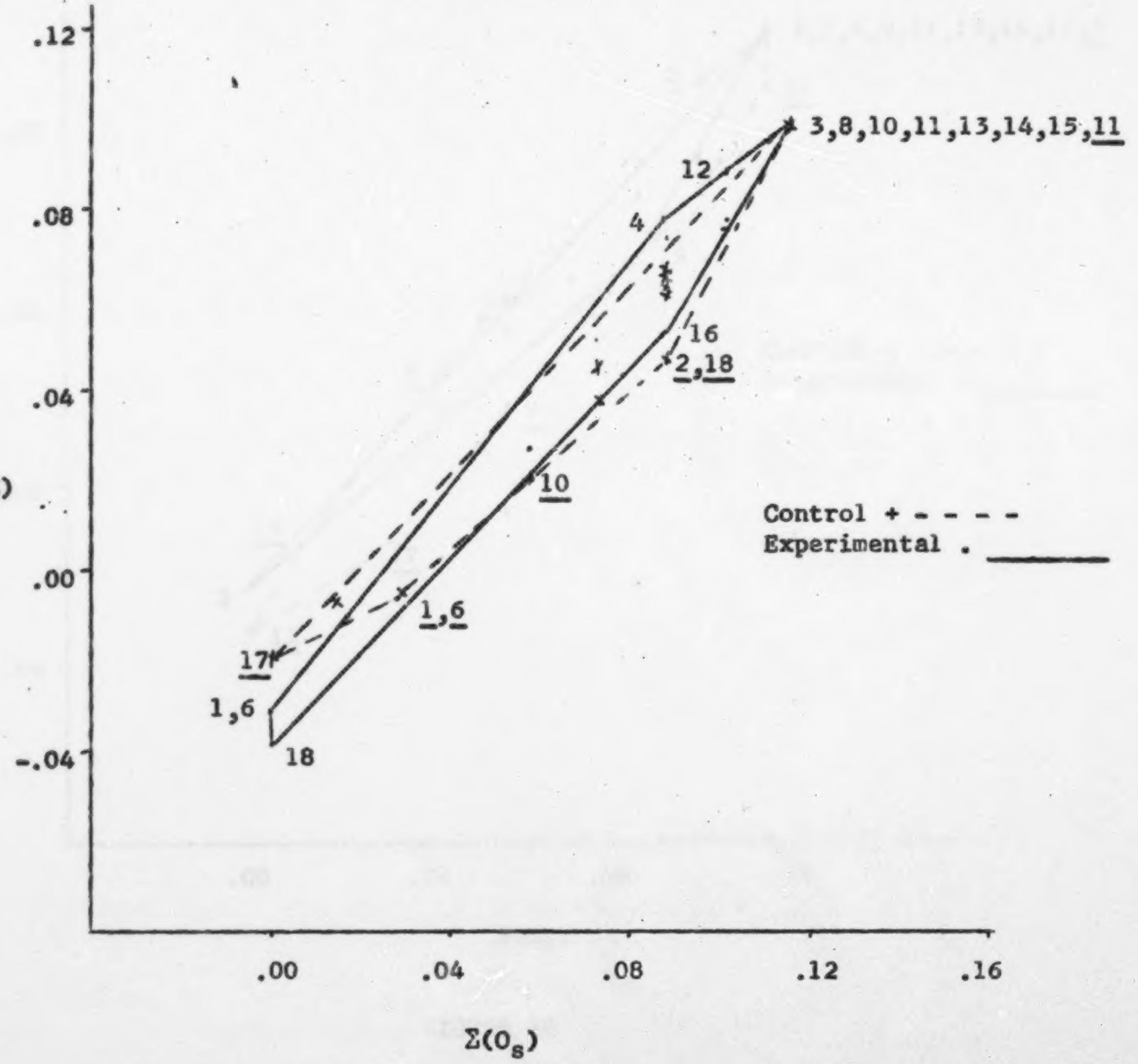

FIGURE 48

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 35B' OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS

Control

Subjects	Tactics
11	5,15,6,13,4
17	2,5,4,6,8,15
1	1,2,3,4,5,6,7,8,9,10
6	2,4,5,11,13,14,16,15,8,6
10	5,7,14,6,2,13,15,11,9,16,12
2	5,2,7,15,11,6,4,8,14,12,13,9
18	5,13,15,16,14,7,8,6,12,9,11,4

Experimental

Subjects	Tactics
3,15	5,6,15,13,4
8,10	5,15,6,4,13
11,13,14	5,6,15,4,13
12	5,2,8,11
4	5,6,11
1	11,14,12,16,7,9,4,5,13
6	7,5,4,6,8,15,2,14,11
18	14,16,12,8,5,6,15,2,7,13,4
16	5,15,7,9,11,13,4,12,14,16

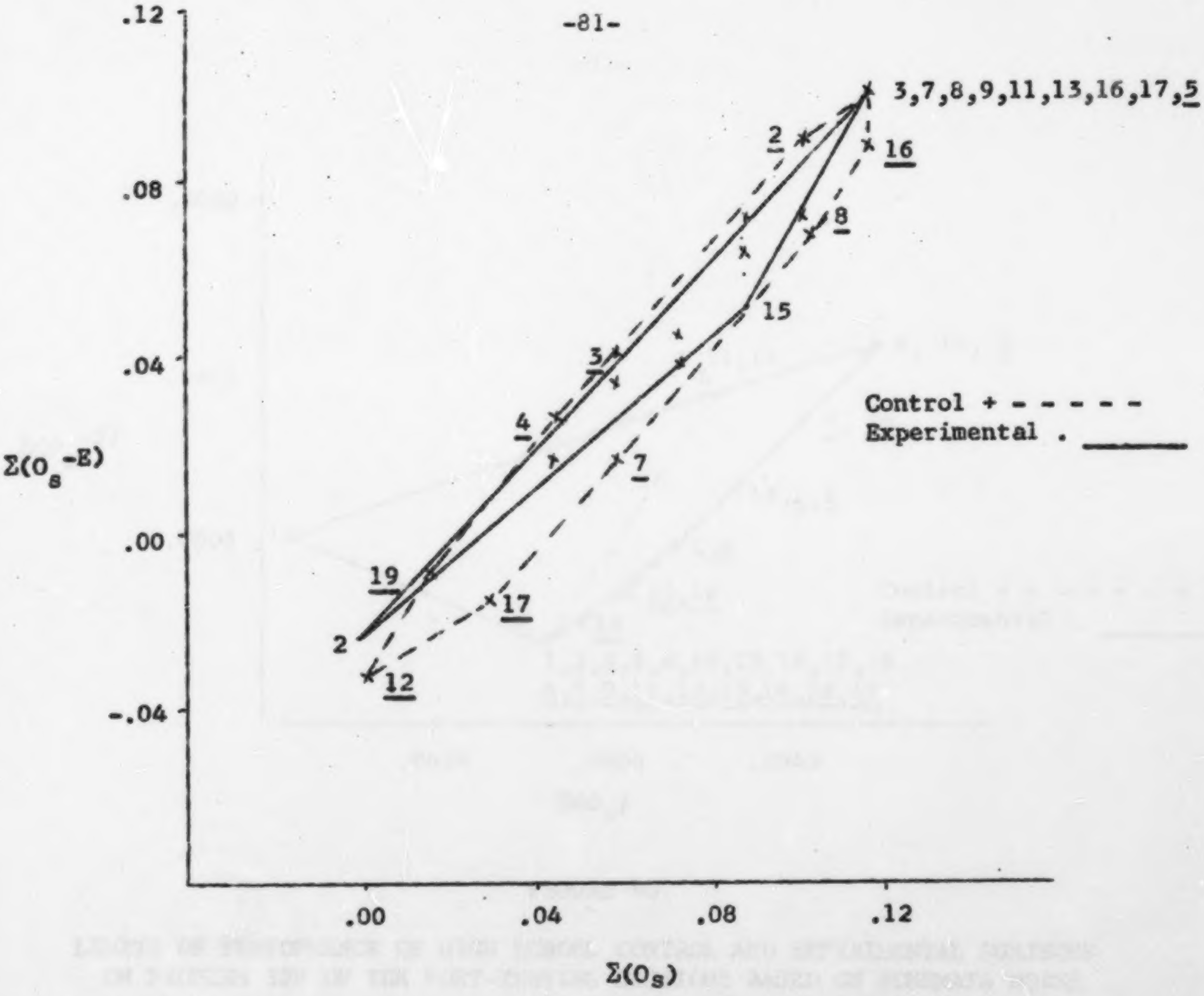

FIGURE 49

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 35B' OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
5	5,2,8,13,4	3,9	5,6,15,4,13
2	5,2,8,13	7,13	5,6,15,13,4
3	5,14,11,2,8	8,12,16,17	5,15,6,4,13
4	15,6,5,13,4	11	5,2,8,14,11
19	7,5,4,8,13,9,3	2	14,11,5,6,15,4,13
12	11,14,9,12,7,13,5,16,4	15	5,15,2,7,6,11,16,8,4,13
17	4,8,15,5,2,13,12,14,11, 6,7,9,16		
7	5,14,11,8,6,15,4,16,12, 9,7,13		
8	5,4,6,11,14,7,9,12,13,16		
16	5,15,6,14,11,8,4,13		

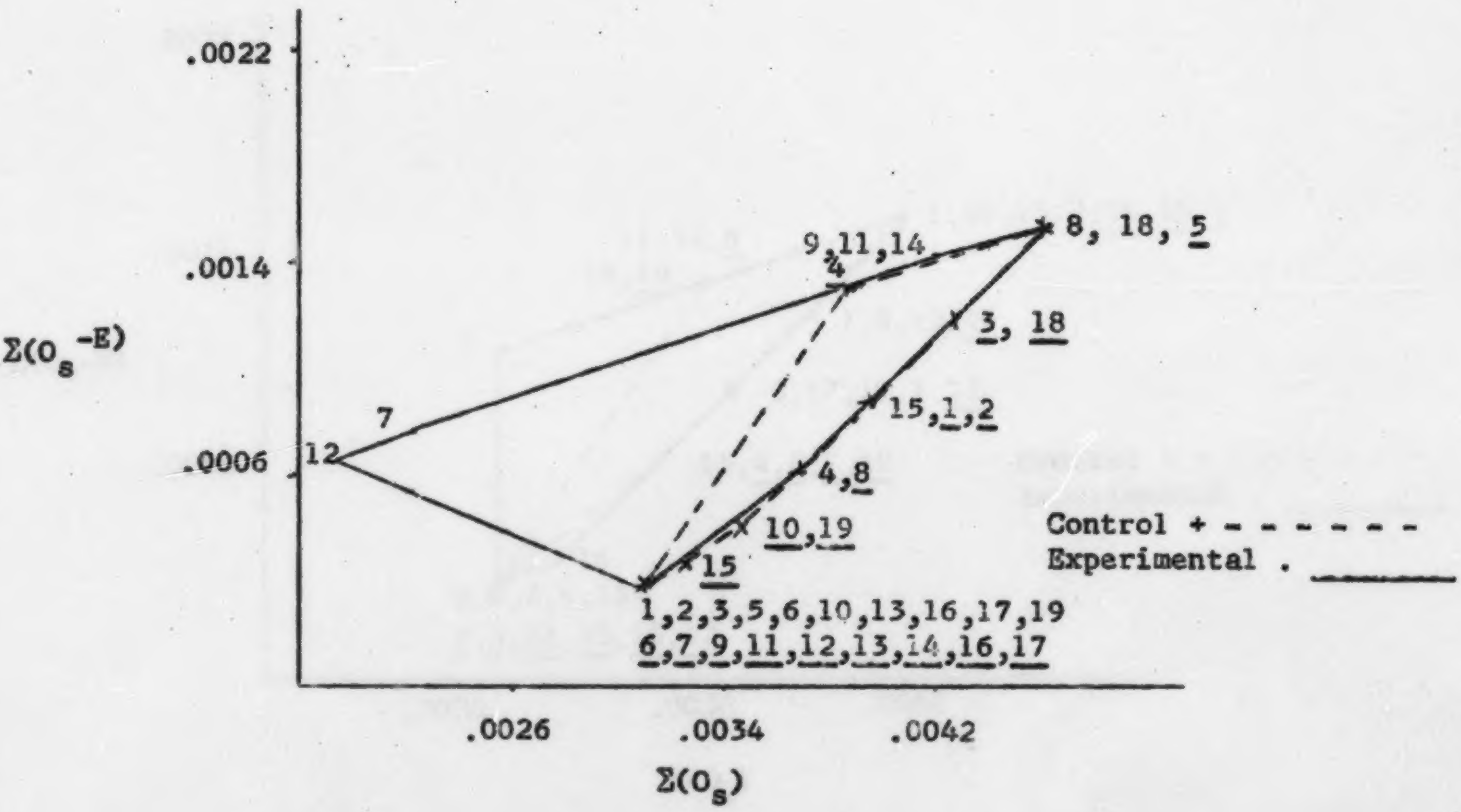

FIGURE 50

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 32F OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS

Control		Experimental	
Subjects	Number of Questions	Subjects	Number of Questions
5	12	8,18	12
4	10	9,11,14	11
6,7,9,11,12,13,14,16,17	18	7	6
15	17	12	5
10,19	16	1,2,3,5,6,10,13,16,17,19	18
8	15	4	15
1,2	14	15	14
3,18	13		

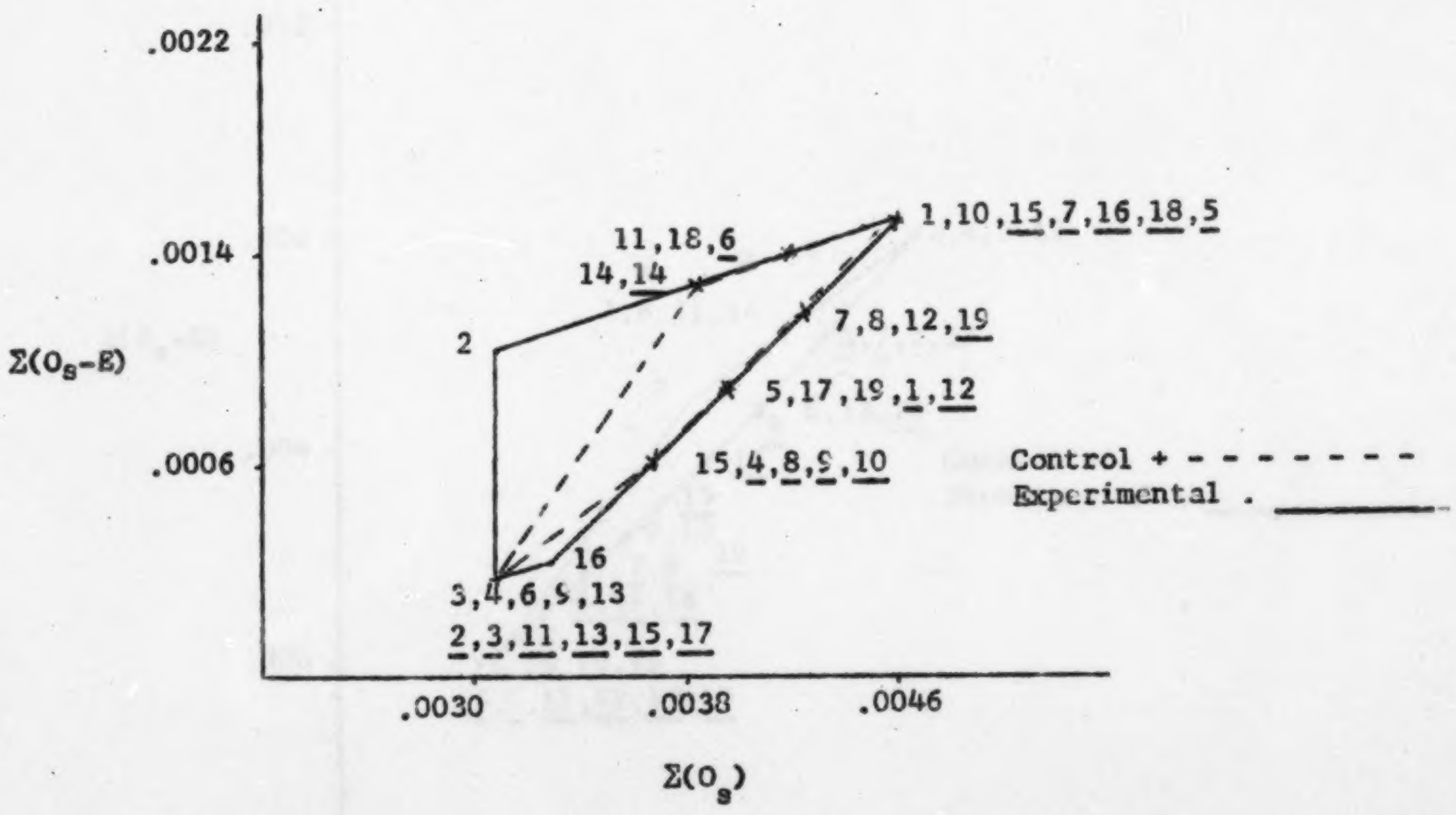

FIGURE 51

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 32F OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control		Experimental	
Subjects	Number of Questions	Subjects	Number of Questions
5,7,15,16,18	12	1,10	12
6	11	11,18	11
14	10	14	10
2,3,11,13,15,17	18	2	8
4,8,9,10	15	3,4,6,9,13	18
1,12	14	16	17
19	13	15	15
		5,17,19	14
		7,8,12	13

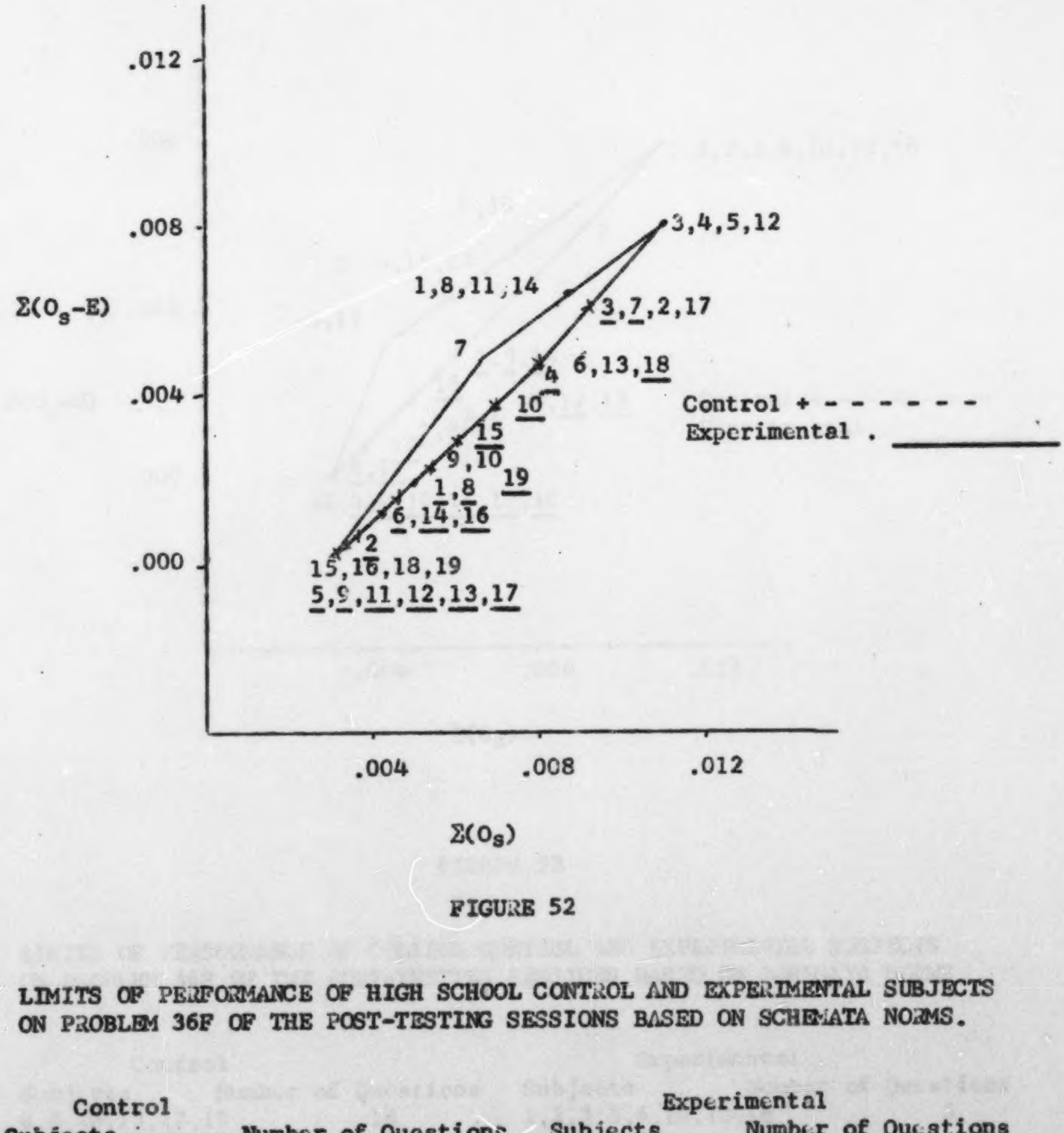

FIGURE 52

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 36F OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control		Experimental	
Subjects	Number of Questions	Subjects	Number of Questions
5,9,11,12,13,17	18	3,4,5,12	5
2	17	1,8,11,14	4
6,14,16	16	7	3
1,8	13	15,16,18,19	18
19	12	9,10	11
15	10	6,13	7
10	9	2,17	6
4	8		
18	7		
3,7	6		

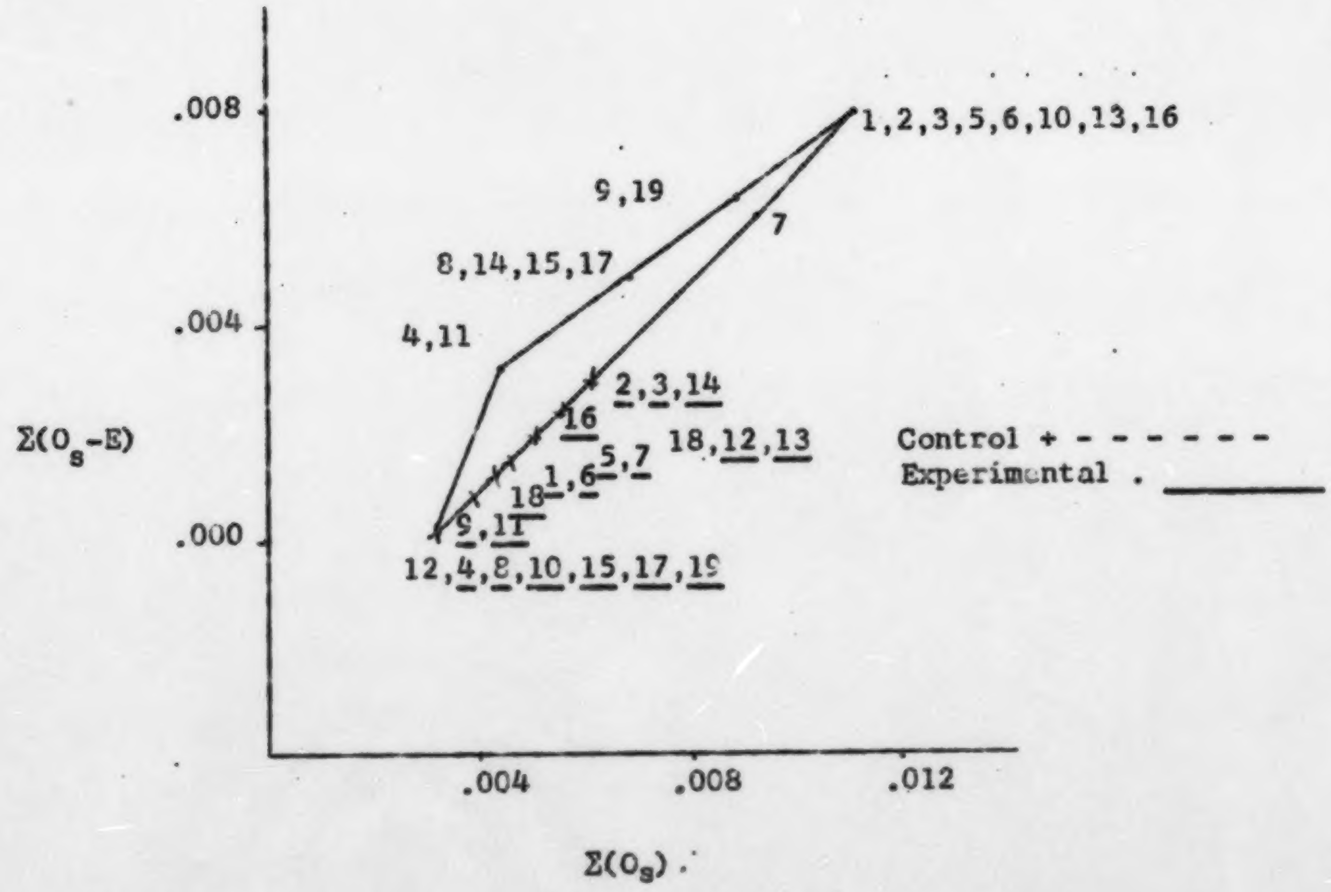

FIGURE 53

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 36F OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control		Experimental	
Subjects	Number of Questions	Subjects	Number of Questions
4,8,10,15,17,19	18	1,2,3,5,6,10,13,16	5
9,11	17	9,19	4
18	14	8,14,15,17	3
1,6	13	4,11	2
5,7	12	12	18
12,13	11	18	11
16	10	7	6
2,3,14	9		

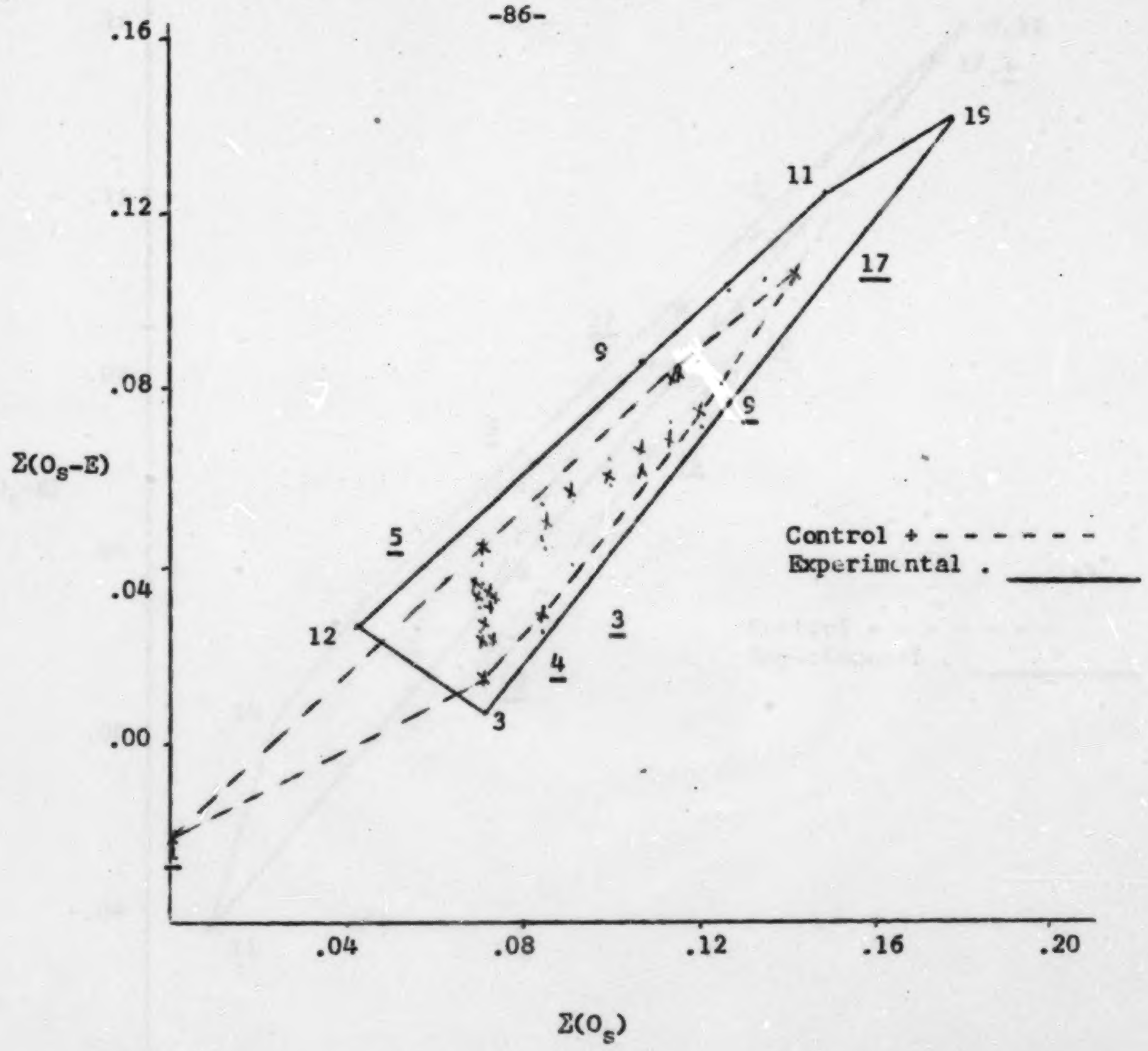

FIGURE 54

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 26 OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
17	1,2,6,7,8,9,5	19	1,7,9,6,8,13,5
5	1,2,3,4,7	11	1,6,7,8,9
1	12,10,5,11	9	1,9,6,3,13
4	1,2,3,4,11,12,13,9,6,7,8	12	6,7,8
3	1,2,3,4,9,13,7,8,6,14,12	3	1,2,3,4,7,6,9,13,5,14,11,10,8
9	1,9,7,2,3,4,12,11,10		

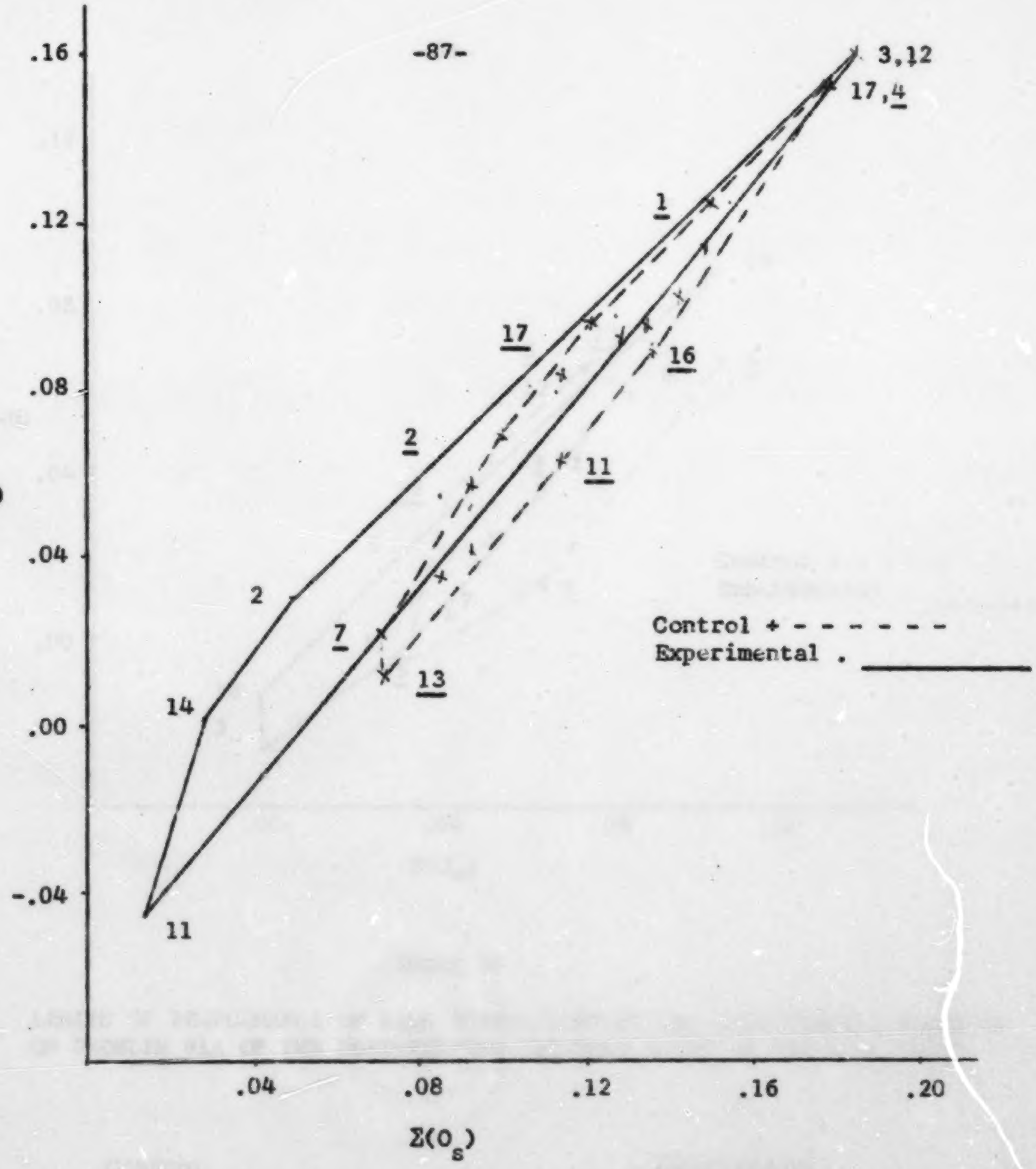

FIGURE 55

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 26 OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control		Experimental	
Subjects	Tactics	Subjects	Tactics
4	1,7,9,6,8	3,12	1,9,7,6,8
1	1,6,7,8,9	2	9,7,3,6
17	1,9,8,3,6	14	7,14,13,8,9,2
2	1,3,8,2,9,6	11	6,10,11,8,12,14,3,7,13,5,4,9
7	1,10,11,12,2,4,3,6,7,8		
13	1,10,11,12,2,3,4,6,5,9,7,8	17	1,7,6,9,8
11	1,6,11,9,7,4,3,8,10,2		
16	1,6,7,4,9,3,10,8,5		

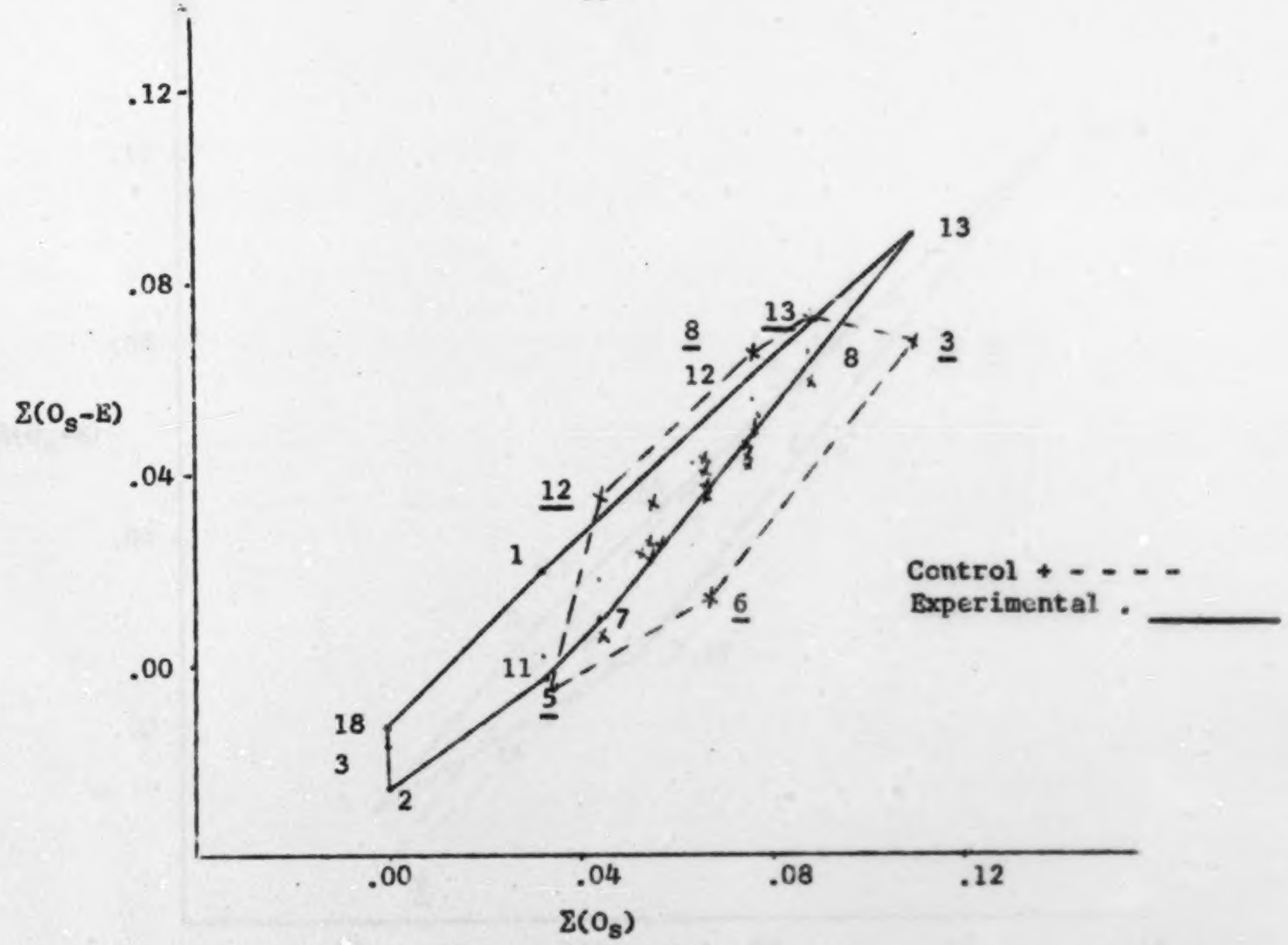

FIGURE 56

LIMITS OF PERFORMANCE OF HIGH SCHOOL CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 41A OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control

Subjects	Tactics
13	1,5,6,3
8	1,5,6
12	1,5
5	15,13,4,1,3,6,14,8,5
6	1,4,6,10,11,12,3,5,15,14, 13,2
3	1,5,6,11,3,15,13,4,8,9

Experimental

Subjects	Tactics
13	1,9,6,5,3
12	1,5,3,9
1	1,15,13
18	6,9,15
3	11,4,3,13
2	6,3,13,10,8,11
11	14,8,4,1,6,9,3,5
7	8,4,3,9,11,5,10,13
8	14,5,6,3,10,9

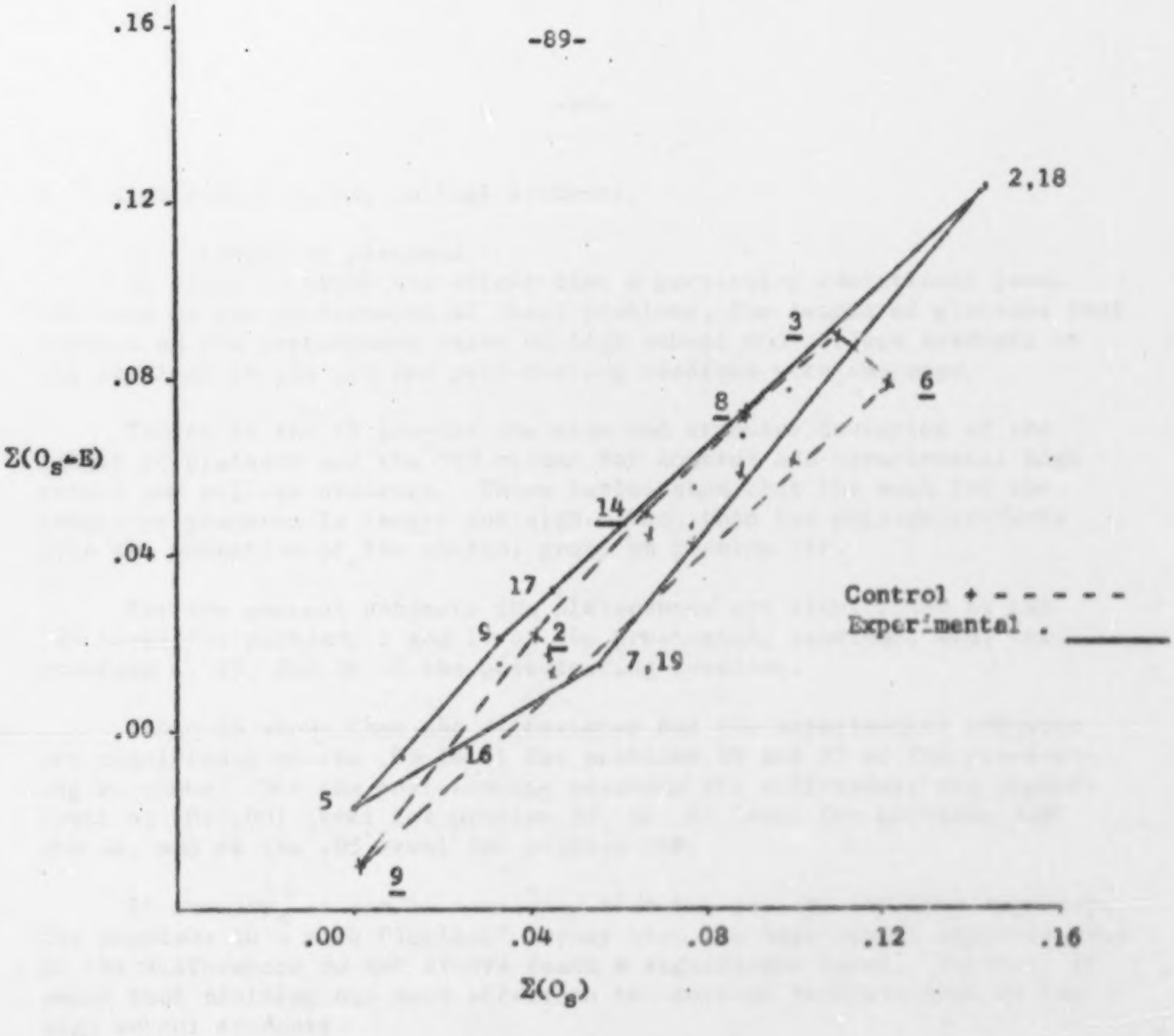

FIGURE 57

LIMITS OF PERFORMANCE OF COLLEGE CONTROL AND EXPERIMENTAL SUBJECTS ON PROBLEM 41A OF THE POST-TESTING SESSIONS BASED ON SCHEMATA NORMS.

Control			
Subjects	Tactics	Subjects	Tactics
3	1,6,9,3,5	2,18	1,5,6,9,3
8	1,5,6,3	17	10,6,9
14	1,3,9,6	9	3,6,5
2	6,3,5,9	5	6,3,8,13
9	5,14,13,8,6,9,3	16	5,6,7,10,1,9
6	14,1,6,10,8,3,11,2,13,5	7	1,6,3,2,13,8,11,5,9
		19	5,6,13,1,3,14,4,8,10

2. High school versus college students.

a) Length of plateaux.

In order to study the effect that a particular educational level may have on the performance of these problems, the length of plateaux that appears on the performance curve of high school and college students on the problems of the pre and post-testing sessions were compared.

Tables 14 and 15 present the mean and standard deviation of the length of plateaux and the "t" values for control and experimental high school and college students. These tables show that the mean for the length of plateaux is larger for high school than for college students with the exception of the control group on problem 36F.

For the control subjects the differences are significant at the .05 level for problems 1 and 19 of the pre-testing sessions, and, for problems 1, 25, and 26 of the post-testing sessions.

Table 15 shows that the differences for the experimental subjects are significant at the .05 level for problems 19 and 25 of the pre-testing sessions. For the post-testing sessions the differences are significant at the .001 level for problem 25, at .01 level for problems 31D' and 26, and at the .05 level for problem 36F.

In summary, it can be concluded that the college students approach the problems in a more "logical" manner than the high school students even if the differences do not always reach a significant level. Further, it seems that training has more effect on the college students than on the high school students.

TABLE 14

MEAN AND STANDARD DEVIATIONS OF LENGTH OF PLATEAUX, NUMBER OF SUBJECTS AND "t" VALUES FOR EACH ONE OF THE PROBLEMS OF THE PRE-TESTING AND POST-TESTING SESSIONS ON THE HIGH SCHOOL AND COLLEGE CONTROL SUBJECTS.

Problems		High School			College			
		M	σ	N	M	σ	N	"t" Values
Pre-testing	1	1.95	1.54	19	.95	1.00	19	2.37*
	19	5.05	3.30	19	3.37	2.41	19	1.79*
	25	6.42	4.61	19	5.16	3.12	19	.99
Post-testing	1	.84	.87	19	.32	.57	19	2.18*
	19	2.79	1.91	19	2.47	1.98	19	.51
	25	4.37	2.68	19	3.11	1.59	19	1.76*
Type a	31D'	3.95	2.09	19	3.37	2.52	19	.77
	35B'	5.68	2.51	19	5.05	3.14	19	.68
Type b	32F	4.00	2.25	19	2.79	2.44	19	1.58
	36F	8.58	4.48	19	9.21	3.46	19	-.49
New Problems	26	6.00	2.20	19	4.47	2.85	19	1.85*
	41A	4.26	2.59	19	3.47	2.28	19	1.00

*P∠05

TABLE 15

MEAN AND STANDARD DEVIATIONS OF LENGTH OF PLATEAUX, NUMBER OF SUBJECTS AND "t" VALUES FOR EACH ONE OF THE PROBLEMS OF THE PRE-TESTING AND POST-TESTING SESSIONS ON THE HIGH SCHOOL AND COLLEGE EXPERIMENTAL SUBJECTS.

		High School			College			
Problems		M	σ	N	M	σ	N	"t" Values
Pre-testing	1	1.58	1.84	19	1.05	1.32	19	1.02
	19	4.42	2.26	19	3.00	1.62	19	2.23*
	25	5.37	2.64	19	3.53	2.14	19	2.36*
Post-testing	1	.95	1.70	19	.37	.74	19	1.36
	19	2.95	2.46	19	1.84	1.56	19	1.66
	25	5.21	2.80	19	2.74	1.33	19	3.47***
	31D'	2.21	1.91	19	.95	.94	19	2.58**
Type a	35B'	3.05	3.73	19	1.74	2.51	19	1.27
	32F	3.42	2.82	19	2.47	2.44	19	1.11
Type b	36F	3.68	5.13	19	1.05	3.08	19	1.92*
	26	5.32	3.14	19	3.00	2.70	19	2.44**
	41A	3.47	1.76	19	2.42	2.09	19	1.68

*P<.05
**P<.01
***P<.001

b) Convex sets

Comparing the convex sets for high school and college students for problem 1 of the pre-testing sessions (figures 34 and 35) show that the convex sets for the college students are higher than the convex sets for the high school students. On the same problem for the post-testing sessions (figures 40 and 41) the convex sets for college students show less dispersion than the convex sets for high school students.

In problems 19 and 25 of the pre and post-testing sessions (figures 36 to 39 and 42 to 45) the convex sets for college students show less dispersion than the convex sets for high school students.

In problem 31D' the convex sets for college experimental subjects (figure 47) show less dispersion than the convex sets for the high school students (figure 46). Further, 17 college experimental subjects have the higher (O_s) score whereas only 11 high school experimental subjects have the higher (O_s) score. On the convex sets for control subjects not much differentiation is found.

The convex sets for problems 35B', 32F and 36F show little differentiation between high school and college subjects (figures 48,49,50, 51,52 and 53).

The convex sets for problem 26 (figures 54 and 55)show larger dispersion in terms of the ($O_s - E$) values for the experimental college subjects than for the high school experimental subjects. Comparing the control subjects the convex sets for the high school students shows more dispersion than the convex sets for the college students.

The convex sets for problem 41A (figures 56 and 57) show larger values for college than for high school students in both control and experimental subjects. As a general statement it can be said that the college students show "better" performance than the high school students on approaching these problems.

CHAPTER V

SUMMARY AND CONCLUSIONS

This research has been designed in order to study individual differences in thought processes. Its primary interest was to study the process followed by the subject in order to reach the solution of the problem. For this purpose the performance of 38 experimental subjects were studied throughout 24 problems of the training sessions.

The methods used to perform the study of this first part of the research were: group norms, length of plateaux calculated from the schemata norms, and performance curves based on group norms and on schemata norms.

Of special interest was the study of the complexity of the problems represented by the schemata and the degree of abstraction of the content. The analysis of variance using group norms and length of plateaux show that the main effect schemata and the main effect content are statistically significant. The interaction between schemata and content is also significant.

When comparing the performance curves based on group norms with the ones based on schemata norms, it is clear that the latter gives more useful information about the process followed by the subject when he is solving a problem.

The second aim of this research was to study the individual differences in the process of solving a problem between subjects with training and subjects without training. For this purpose, the performances of 38 experimental subjects were compared with the performances of 38 control subjects individually matched before the experiment. These comparisons were made on the basis of 12 problems, 3 pre-testing and 9 post-testing.

The methods used to measure their performance were: schemata norms, length of plateaux, individual performance curves based on schemata norms, and convex sets based on schemata norms. The experimental subjects show a "better" performance than the control subjects. When the experimental subjects had any plateaux at all, they were shorter than the plateaux of the control subjects.

The individual performance curves show that the process followed by the experimental subjects in order to solve a problem is "better" than the one followed by the control subjects. This means that the experimental subjects always approach the problems in a more "logical" way. The greatest differences were found in the problems similar to those used in the training sessions.

The third aim of this research was to study whether the educational level had an influence on the process followed by a subject in order to solve this type of problems. The study of the length of the plateaux shows clearly that the process followed by the college students is always "better" than the one followed by the high school students.

The results of the present research confirm the one obtained on the previous one (Rimoldi, Fogliatto, Haley, Reyes, Erdmann, Zacharias, 1962). The control and experimental subjects were compared on the number of questions they used in order to solve the problems and compared on the agreement concerning the questions they should ask. The results of the comparison between college and high school students is also confirmed.

The methods used in the present research are more sensitive than the methods used on the previous research. The schemata norms give more clear information on the process followed by a subject in order to solve a problem than the group norms. More sensitive even than the study of the accumulative score according to schemata norms is the study of length of plateaux.

Plateaux are found in that stage of a subject's performance when he asks either irrelevant questions or relevant questions out of their proper order. Thus, the length of a plateau is measured by the number of irrelevant or out-of-order questions selected in sequence. This measure may be interpreted in several ways. One would simply be an indication of the subject's lack of progress toward a solution. It might also be a period in which the subject is reformulating the problem. Likewise, it could merely represent a type of non-good-directed behavior during which the subject is "groping" for a possible clue. Regardless of the interpretation, the length of plateaux has shown itself to be an effective measure in the characterization of process.

The study of the convex sets using schemata norms also give clear information on the process followed by a subject in solving a problem, and also permits one to compare two groups of subjects.

PHASE II

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PHASE II

CHAPTER VI

PROCEDURE

Table 16 presents the experimental design corresponding to Phase II. The total number of subjects used was forty - half of which were high school students, and half of which were college students. To each of these subjects, twenty-two problems were administered, eleven of which were selected from our previous research (CRP No. 1449) and from Phase I of this study. The remaining eleven problems were devised to test some of the findings obtained previously.

TABLE 16

EXPERIMENTAL DESIGN

RAVEN'S PROGRESSIVE MATRICES
TEST AND THOUGHT PROBLEMS

		HIGH		LOW		
		High School	College	High School	College	
Academic Grades	High	5*	5	5	5	
	Low	5	5	5	5	
						40

* The number in each cell represents the number of subjects in each group.

Subjects

The subjects used in this study were all males selected on the basis of two major criteria: a) academic grades and, b) performance in both the Standard Progressive Matrices, 1938 by J. C. Raven, and Thought

Problems by Tate, Stanier and Harootunian. Both the high school and college samples were classified into four subgroups, I, II, III and IV representing respectively: high academic grades and high performance in the Raven's Test and the Thought Problems, high academic grades and low performance in the tests, low academic grades and high performance in the tests, and low academic grades and low performance in the tests.

a) High School Subjects

The school records of 540 male students just entering the freshman year of high school were analyzed. Since the selection took place prior to any academic evaluation of these students in high school, the analysis was performed on the SRA High School Entrance Examination records of the 540 male students. On this basis, eighty students were selected, forty having the highest scores of the 540, and forty having the lowest scores. To these eighty students, the Raven's and the Thought Problems (Part I)* tests were administered in group form.

The joint analysis of these results made possible the differentiation of the four categories, I, II, III and IV, used in the design. When the grades for the first semester became available, the average grade for each of the selected students was calculated. These averages were based only on the courses that were common to all the students. These were: English, Algebra, Mechanical Drawing and Religion. The members of the high group also pursued Latin, Biology and a laboratory course in Biology, those of the low group, besides the common courses, took Woodshop and General Science.

The averages of the academic grades for the high and low groups are respectively 85.35 and 72.28, with the corresponding standard deviations 7.36 and 7.88. The difference between the two means is significant beyond the .01 level; that is, the I + II and III + IV groups are clearly separated in terms of their school grades. Ten subjects with high academic standing were assigned in equal number to Groups I and II on the basis of their performance in the Raven and Thought Problems Tests. A similar procedure was followed with the ten subjects with low academic standing, obtaining finally five subjects in each of Groups I, II, III and IV.

In Table 17, the mean and standard deviation for both academic grades and test performances are given for Groups I, II, III and IV. On the extreme right-hand side of this table, the mean and standard deviation of the average academic grades are presented for the high and low academic groups, respectively. At the bottom, the mean and standard deviation of test scores are presented for subjects classified as high and low in test performance. Notice that subjects belonging to Groups I and III have higher means in both Raven and Thought Problems than subjects belonging to Groups II and IV. These differences are significant beyond the .001 level.

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* Part I refers to the first half of the test.

TABLE 17

Means and Standard Deviations of Academic Grades, Ravens, and Thought Problems for High School Students

		High in Test Groups			Low in Test Groups			Combined Groups		
	Source	Group	Mean	σ	Group	Mean	σ	Group	Mean	σ
High Academic Groups		I			II			I+II		
	Academic Grades		89.25	6.54		81.45	5.92		83.5	7.36
	Raven		55.80	3.12		40.60	3.88			
	Thought Problems		17.00	1.74		8.50	2.97			
Low Academic Groups		III			IV			III+IV		
	Academic Grades		77.60	5.13		66.95	6.41		72.28	7.88
	Raven		48.00	3.63		35.20	1.94			
	Thought Problems		11.30	1.91		5.30	1.21			
Combined Groups		I+III			II+IV					
	Raven		51.90	5.17		37.90	4.08			
	Thought Problems		14.15	3.62		6.90	2.77			

Comparing the academic grades for the four groups, I, II, III and IV, it is seen that there is a decrement in the corresponding means for the four groups in the order stated; the differences between the means are significant at least at the 5% level in all cases with the exception of Groups II and III, and II and I. For the Thought Problems Test, the means, when analyzed in decreasing order of magnitude, follow the sequence I, III, II and IV, as expected. The differences are significant at least at the .01 level with the exception of Groups II and III, and II and IV.

In the case of the Raven's Test, the means follow the same order as in the Thought Problems, all the possible differences being significant at least at the 5% level.

Groups I and IV are clearly differentiated in all the tests and grades used for identifying the groups. On the other hand, Groups II and III are only significantly different (.05 level) in terms of the Raven's Test. The crucial comparison in terms of our design refer to Groups I and IV and II and III. While Groups I and IV have been clearly differentiated, II and III overlap in terms of grades and Thought Problems.

In summary: 1) in terms of academic grades, the selection of a low and high group has been successful; 2) in terms of test performances the selection of a high and a low group has also been successful; 3) Groups I and IV, which represent the two extreme poles of the academic grades and test performance continua are clearly separated; 4) there is an overlap between the intermediate Groups II and III.

On the basis of the design, there should be a nonsignificant correlation between academic grades and performance in the tests for the subjects of the high academic group as well as for those in the low academic group. The multiple correlations between grades with Thought Problems and Raven's Test scores are .686 and .687, respectively for the high and low academic groups. With a sample of the size used here, these are not significant (at least at the .05 level).

The average age for the selected group is 14.25 years with a standard deviation of .536.

b) College Students

From 200 male students in the freshman class of the college of arts and sciences at a university, 114 were contacted during their second semester in college. They were selected on the basis of their average grades for the first semester, so that approximately half had a grade point average of 3 or more, and half had a grade point average smaller than 2.* After administering the Raven's Test and the Thought Problems, a sample of twenty subjects was chosen so that Groups I, II, III and IV with specifications similar to those of the high school sample could be defined.

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* The grade point average is based on a four point system.

In Table 18, means and standard deviations are given for each of the I, II, III and IV groups as well as for the high and low academic, and for the high and low test score groups.

The differences between the means for academic grades in the high academic (I and II) and low academic (III and IV) groups are significant beyond the .001 level as expected. Similarly, the difference of the mean in test scores between the high score (I and III) and low score (II and IV) groups is significant beyond the .001 level for both the Raven's and Thought Problems tests. The comparison between the I and IV groups for all the variables used to characterize them are all significant beyond the .001 level in the expected direction. Between the II and IV group the differences are only significant in terms of academic grades (beyond the .001 level) in the expected direction. However, the differences between the means for the Raven's Test and the Thought Problems are not significantly different (at least at the .05 level) between the II and III groups. Again, as in the case of the high school students, there is an overlap between the II and III groups.

The multiple correlations between grades with Thought Problems and Raven's Test scores are .670 and -.240 respectively for the high and low academic groups. These values are not significant at least at the .05 level.

The average age for the college group is 18.40 years with a standard deviation of .490.

c) Problems

Eleven problems, developed in previous research, were used in this study. These are 31A, 31B, 31C, 31D, 35A, 35B, 35C, 35D, 36C, 36D, and 25. The rationale underlying these problems, their manner of presentation and their characteristics have been described in great detail in CRP No. 1449 (Rimoldi, Fogliatto, Haley, Reyes, Erdmann, Zacharia, 1962) and consequently will not be discussed here. Suffice it to mention that in a few instances, slight modifications were introduced in the problems to make their representation less ambiguous. The interested reader will find the pertinent material in the Appendix. The reasons for these changes emerged from the experience and results obtained in CRP No. 1449 and in Phase I of this study. For instance, instructions were given specifying that the subject should not ask more questions than those needed. In the older form, subjects could generate their own questions in solving problems 36C and 36D; while in this study, the subject had to choose among eighteen possible questions. In problems 31B, 31C, 31D, 35B, 35C, and 35D, minor changes were made in presentation. The questions and answers were not changed. All the eleven new problems have ten possible questions. This was done in order to simplify scoring procedures. For instance, the total number of conceivable sequences will be the same for all of them.

TABLE 18

Means and Standard Deviations of Academic Grades, Ravens, and Thought Problems for College Students

		High in Test Groups			Low in Test Groups			Combined Groups		
	Source	Group	Mean	σ	Group	Mean	σ	Group	Mean	σ
High Academic Group		I			II			I+II		
	Academic Grades		3.34	.22		3.13	.20		3.24	.24
	Raven		58.60	.80		53.40	1.85			
	Thought Problems		18.30	.60		14.10	2.01			
Low Academic Groups		III			IV			III+IV		
	Academic Grades		1.55	.23		1.59	.34		1.57	.29
	Raven		54.80	1.83		48.80	2.40			
	Thought Problems		15.80	.81		10.70	2.40			
Combined Groups		I+III			II+IV					
	Raven		56.70	2.37		51.10	3.14			
	Thought Problems		17.05	1.44		12.40	2.79			

Subjects could not generate their own questions, but had to choose among those given with the problem. An attempt was made to build instruments such that one or two sequences of questions would give the subject all the information needed to solve the problem. As it will be seen by analyzing the results, this effort was more successful in some problems than in others. The new problems are presented in the Appendix and bear the following code numbers: 40, 42, 44A, 44B, 44C, 44D, 46, 59A, 59B, 59C, 59D; excepting problems 59A, 59B, 59C and 59D, all are figure problems. All the problems of 44 series have essentially the same figure; the major differences are due to the content used. For instace, in Figure 44A, the areas are designated by numbers, and the questions that may be asked refer to the perceptual characteristics of these areas, (color, type of border, etc.). In Figure 44B, the areas are identified by numbers. The questions that the subject may ask are concerned with these numbers. Nevertheless, there is not a one to one relationship between these numbers and the boundaries of the areas, and this will be irrelevant in solving the problem. Problem 44C is similar to 44A except that the questions are worded negatively. In Problem 44D, areas are identified by combinations of letters. Though there is a relationship between boundaries and letters, the solution does not hinge upon this fact as it can be seen by studying the questions that the subject may ask. In order to establish the "best" sequences, the properties of the problems were carefully analyzed. The following sequences were decided upon:

Problem	"Best" Sequences
44A	7, 5, 9
44B	6, 4, 8
44C	1, 7, 10
44D	5, 7, 9

In establishing these sequences, the principle of proceeding to questions that five progressively less information was adopted. Take the case of Problem 44A. After analyzing the question, the possibility that question 3 should be asked first is quete compelling since it will be eliminating half of the areas. However, some of the information obtained by asking question 3 first can be secured by a close inspection of the figure and of the questions provided with the problem. That is, we are assuming that subjects will carefully inspect figures and questions before attempting to solve the problem. As it will be seen later, this seems to have been supported by the actual protocols. A similar reasoning may be used to explain Problems 44B, 44C and 44D.

Concerning the "best" sequence to be used to solve Problem 40, the same type of assumption was made: that subjects proceed first by inspecting the figure and the questions and then by asking some questions that would eliminate the maximum number of areas at a given stage in the process of solution. The sequence decided upon was 2, 7. For Problem 42, the tactic 3,1,5,8 corresponds to the sequence used to score subjects. By tactic, we understand the sequence of questions asked by a subject.

In Problem 46, the sequence decided upon is 2,7,8,9. In all these problems, the subject had to identify one area. Notice that in the four problems of Series 44, the differences are related to the content used; the number of areas in all of them being 16.

In terms of information theory, four questions should be required to identify the area. Nevertheless, the information provided by the perceptual characteristics of the figure make it possible to reach the solution asking fewer than four questions. In the case of Problem 40, which presents twenty areas, the solution should be obtained after asking five questions (in terms of information values, 4.32193 bits are needed). Nevertheless, an examination of the figure and of the questions makes it possible to identify the crucial area by asking only two questions. Notice that Problem 46 also consists of twenty areas, and theoretically has the same information value as Problem 40. Nevertheless, here four questions are needed for the solution. Thus, the comparison between the performance in these two problems will give clues as to the information value inherent in the perceptual characteristics of the problem and of the questions.

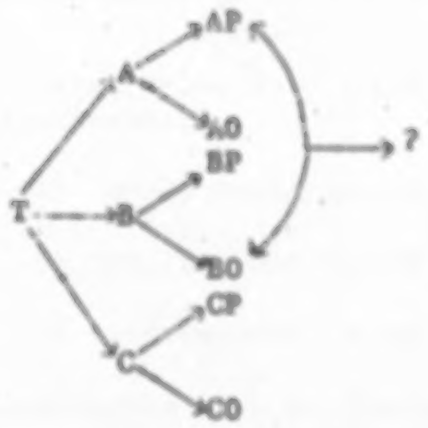
Problems 59A, 59B, 59C and 59D were built by deciding first on a logical framework that can be most conveniently represented by the tree shown in Figure 58. Other representations could, of course, be made.

Essentially, in all problems of series number 59, the subject had to identify the values corresponding to two of the terminal branches of the tree as presented. But, in order to perform this operation, it was necessary because of the questions given, to perform subtractions and to combine values. Since these could be performed in several ways, and since it would be logically impossible to weigh them preferentially, the scoring norms consider several sequences together. Since these final sequences are based on the schema of the problem, they will be called schemata norms.

In terms of the tree, all these problems are similar, but because of the different languages used (content) the problems are not strictly parallel. For instance, in form 59D, presented in an algebraic content, without numbers involved, the answer to a question may provide more information than the same answer value given in numbers. Elements impossible to identify with numerical answers become distinct when using algebraic symbols.

In Problem 59A, the wording was such that an ambiguity may arise in terms of whether the required solution supplies only the combination of two branches or the identification of the two branches of the tree separately. This became obvious late in our research. The group performance values were analyzed in order to establish how the subjects interpreted the problem. Unfortunately, this did not clarify the issue and thus Problem 59A was dropped from further consideration.

FIGURE 58



Problems of series 59.

On the basis of our previous studies, problems of series 31 and 35 were included in this research. Problems of series 59 are logically more complex in terms of the classification and relationships involved in the schema. For instance, problems of series 35 (CRP No. 1449, Rimoldi, Fogliatto, Haley, Reyes, Erdmann, Zacharia, 1962) involve two trichotomies, but the answer required only identifying one of the final branches. Furthermore, the questions that the subjects may ask were presented in such a fashion that the number and type of operations to be performed did not require the complexity involved in problems of series 59.

d) Administration of the Problems

The high school subjects were examined in the school. The Raven's Test and the Thought Problems were administered in group form.. The same type of administration was used with the college students. The size of the group varied slightly in the different sessions.

The problems were administered individually in both high school and college. The order of administration was as follows: 31A, 31B, 31C, 31D, 40, 42, 36C, 36D, 35A, 35B, 35C, 35D, 44A, 44B, 44C, 44D, 59A, 59B, 59C, 59D, 46, 25.

Notice that old and new problems were not given in separate sequences, and four figure problems (40, 42, 36C, 36D, etc.) were interspersed among problems without figures.

Each session lasted for approximately one hour, and several sessions were used for each subject. The total time of each session depended on the subject's willingness to proceed. No more than seven problems were administered in any one session. No time limit was imposed.

CHAPTER VII

Analyses of the Data and Findings

The subjects' performance in all the problems were treated following three major approaches:

a) Group Performances

b) Pulling Out Method

c) Information Curves

These three approaches will be discussed in the stated order.

a) Group Performances

This approach estimates for each problem and for each group of subjects the frequency with which each question was asked in each possible order.

These frequencies correspond to the values observed in two-way entry tables where columns represent questions, and rows, order in the sequence. If the subjects in a group follow exactly the same tactic, then all the cells in the table will present zero entries, with the exception of one cell per row and per column with a frequency equal to the total number of subjects in the group. In fact, the highest possible dependency between questions and order is at the basis of this type of performance; knowing a question, the order of choice is also known, and vice versa, the uncertainty being minimal.

If the subjects in a group perform in such a way that no relationship whatsoever exists between questions and order of choice, then the cell frequencies will be identical throughout the table. No information can be gained in terms of associating a given question with a given order, the uncertainty being maximal.

Seldom, if ever, will all actual performances follow any of the two previously described patterns. In practice, the performance of a group may approach either of the uncertainty levels just defined. This will depend on two major variables that can be experimentally controlled: a) characteristics of the group, b) characteristics of the problems.

Considering the logical structure of the problem, the language used, the number and wording of the questions and of the corresponding answers, etc., it is possible to state for each problem the sequence or sequences of questions that as a result will represent the maximum empirical assiciation between question and order of choice. This set of sequences corresponds to the schemata norms. In this sense, schemata norms represent the lower empirical limit of uncertainty based on a logical analysis of the problems. This shall be considered to be the criterion of minimal uncertainty for a group performance. It is theoretically conceivable that an observed group performance may yield an uncertainty value smaller than that indicated by the schemata, but in such cases, this may be due to guessing, incomplete performance, poorly constructed problems, etc.

The definition of a criterion for maximum uncertainty is more complex. Several hypotheses can be defended of which only two will be discussed. These will be called H_1 and H_2, respectively.

In hypothesis 1, it is assumed that all the subjects will choose all the questions and that this group performance will reflect no association whatsoever between questions and order. In relation to our previous formulation of maximum uncertainty, hypothesis 1 makes possible its application in a given experimental condition.

In hypothesis 2, the assumption of no association between questions and order is maintained, but the following conditions are added: a) that subjects may choose sequences of varying lengths, b) that these sequences of different lengths have the same chances of appearing. The uncertainty value associated with hypothesis 2 will be slightly less that in hypothesis 1 because of the added conditions. Given m possible choices, the number of sequences of choices possible for any r objects chosen from the m will be $\frac{m!}{(m-r)!}$. That is the number of sequences will vary according to the length (r) of the sequence involved. In order to meet the assumption that sequences of different lengths have the same chances of appearing, the values of $\frac{m!}{(m-r)!}$ must be weighted by a value x_1 such that

$$\frac{m!}{(m-r_1)!}\ x_1 = \frac{m!}{(m-r_2)!}\ x_2 = \frac{m!}{(m-r_3)!}\ x_3 = \ldots.$$

where r_1 is a sequence of length m, r_2 is a sequence of length (m-1), and so on, and values x_1, x_2...are the corresponding multipliers.

Consider that in a sequence of length m, there will be m possible ordered choices. (from 1 to m) and that in a sequence of length (m-1) there will be (m-1) possible ordered choices. Therefore, a table can be constructed which will present all the possible sequence lengths and their corresponding orders of choice. Obviously, the chances of any choice being made in any order are identical for every different sequence length.

For instance, for the case of m=4, Table 19 gives the corresponding frequencies.

TABLE 19

Order of Choice

		1st	2nd	3rd	4th
Length of Sequences	4	6	6	6	6
	3	6	6	6	-
	2	6	6	-	-
	1	6	-	-	-
	Σ	24	18	12	6

Thus, as indicated in the bottom row of Table 19, the addition of the values corresponding to a given order of choice will give the frequency with which any question can be chosen in that order in all the sequences in which it may appear.

In order to build hypothesis 2, we have to consider every choice in every order. Thus, when m=4, the results of Table 19 can be rearranged in the following fashion, Table 20.

TABLE 20

Choices (m)

	1	2	3	4
1st Order	24	24	24	24
2nd Order	18	18	18	18
3rd Order	12	12	12	12
4th Order	6	6	6	6

The values of the entries in tables like Table 20 can be readily computed by using the following expression:

$$\left[(m+1)-c_i\right]\ (m-1)!,$$

where c_i indicates order, and $i=1,\dots i\dots m$

Thus, for $m=4$ and $c_i=2$, we obtain

$$[5-2]\ (3)! = 18$$

For different values of m, the uncertainty measures for tables like Table 20 can be computed.

Any observed performance can be then located along a continuum varying from minimum uncertainty, as defined by the schemata, to maximum uncertainty as defined by H_1 or H_2. For each problem, these limits can be assigned without any references to group performance. They will be considered to be inherent properties of the problems and thus help to define the instruments employed in the experiment. This attempt at characterizing instruments without resort to group indexes is a feature which deserves special emphasis.

The uncertainty value $H(x,y)$* was computed for H_1, H_2 and for the schemata in each problem. Further, this value was also calculated for the observed group performances in all the problems. The groups were as indicated in the previous chapter, I, II, III and IV and their combination I and II (high academic grades), III and IV (low academic grades), I and III (high test scores) and II and IV (low test scores). The results are given in Tables 21 and 22 referring respectively to high school and college students.

These uncertainty values should not be interpreted strictly in terms of information theory. As stated before, they serve to characterize the patterns present in the tables. Our discussion will be limited to define trends in these patterns.**

In relation to Tables 21 and 22, it should be noticed that there is not scheme uncertainty value for problems 36C, 36D and 25. Problems 36C and 36D were developed for previous research before the idea of analyzing logical structures was clearly devised. An "a posteriori" analysis of the problems showed that the possibility of developing best tactics would be very complex. Therefore, no attempt was made in this direction. A similar reasoning applies to problem 25.

* $H(xy) = \log_2 n - \frac{1}{n}\sum_{i}^{r} n_i \cdot \log_2 n_i$, where n equals total number of entries in the table and n_i frequencies in each cell.

** In two yet unpublished studies by Rimoldi and Erdmann, and by Rimoldi, exact probability statements have been devised for two way entry tables. From this it is possible to extend the rationale to include tables with the limitations inherent in our data. Application of this technique to our data would require time not available under the conditions of this grant.

TABLE 21

Uncertainty Values $H_{(x,y)}$ For the High School Groups

Problems	I+II	III+IV	I+III	II+IV	I	II	III	IV	Schema	X	H_1	H_2	Schema/x
31A	3.546	4.709	4.077	4.291	3.156	2.874	4.002	4.107	2.252	1.585	6.644	6.426	1.421
31B	4.786	4.409	4.509	4.724	4.202	4.352	3.684	3.932	2.252	1.585	6.644	6.426	1.421
31C	4.698	4.737	4.425	4.972	3.293	4.681	4.022	4.096	2.252	1.585	6.644	6.426	1.421
31D	4.269	4.915	4.390	4.872	3.622	4.111	4.335	4.134	2.252	1.585	6.644	6.426	1.421
40	4.202	4.952	4.000	5.070	2.557	4.404	4.011	4.578	1.000	1.000	6.644	6.426	1.000
42	4.425	5.201	4.080	5.154	1.971	4.563	4.425	4.664	2.000	2.000	6.644	6.426	1.000
36C	6.104	5.794	6.001	6.019	5.029	5.476	5.249	4.939			8.340	8.097	
36D	6.121	5.394	5.413	5.932	4.984	5.383	4.096	4.912			8.340	8.097	
35A	5.160	5.131	5.082	5.467	3.912	4.932	4.632	4.578	3.922	2.322	8.175	7.934	1.689
35B	4.621	5.196	4.319	5.340	2.965	4.757	4.513	4.681	3.922	2.322	8.175	7.934	1.689
35C	4.789	5.176	4.274	5.380	3.483	4.688	4.168	4.802	3.922	2.322-	8.175	7.934	1.689
35D	4.906	5.315	4.397	5.578	4.004	4.783	4.088	4.765	3.922	2.322	8.175	7.934	1.689
44A	4.431	5.263	4.106	5.257	2.975	3.968	3.732	5.019	1.585	1.585	6.644	6.426	1.000
44B	3.754	4.523	3.676	4.756	2.749	3.785	3.432	4.348	1.585	1.585	6.644	6.426	1.000
44C	3.584	4.972	3.970	5.145	2.642	3.784	4.011	4.802	1.585	1.585	6.644	6.426	1.000
44D	3.467	4.489	3.279	4.461	2.705	3.884	3.404	4.478	1.585	1.585	6.644	6.426	1.000
59B	4.737	3.927	4.581	4.938	4.182	4.134	3.823	4.208	2.823	2.322	6.644	6.426	1.216
59C	4.512	4.702	4.672	4.441	4.203	3.689	4.210	4.134	2.722	2.322	6.644	6.426	1.172
59D	4.704	4.637	4.629	4.890	4.134	4.335	4.055	4.244	3.039	2.322	6.644	6.426	1.309
46	4.152	4.946	3.970	4.938	3.042	3.882	3.936	4.595	2.000	2.000	6.644	6.426	1.000
25	5.005	5.329	5.031	5.318	4.134	4.264	4.323	5.010			9.401	9.148	

TABLE 27

Uncertainly Values $H_{(x,y)}$ For the College Groups

Problems	I+II	III+IV	I+III	II+IV	I	II	III	IV	Schema	X	H_1	H_2	Schema/X
31A	3.255	3.340	3.518	3.314	3.140	2.790	2.816	3.182	2.252	1.585	6.644	6.426	1.421
31B	3.696	4.494	4.128	4.536	2.680	3.849	4.133	3.887	2.252	1.585	6.644	6.426	1.421
31C	3.619	4.463	3.597	3.394	2.606	3.684	3.684	4.282	2.252	1.585	6.644	6.426	1.421
31D	3.212	4.354	4.073	3.816	2.499	2.816	3.974	3.971	2.252	1.585	6.644	6.426	1.421
40	3.679	3.160	4.377	4.773	3.392	2.953	4.431	4.582	1.000	1.000	6.644	6.426	1.000
42	3.711	4.866	2.878	4.738	2.361	4.002	2.919	4.476	2.000	2.000	6.644	6.426	1.000
36C	5.730	5.545	5.397	5.878	4.911	4.875	4.252	5.335			8.340	8.097	
36D	4.735	5.867	4.953	5.709	4.037	3.913	4.533	5.306			8.340	8.097	
35A	3.554	4.980	3.824	4.963	3.099	3.628	3.455	4.718	3.922	2.322	8.175	7.934	1.689
35B	3.689	4.627	3.293	4.773	2.872	3.750	3.426	4.681	3.922	2.322	8.175	7.934	1.689
35C	2.972	4.818	3.729	4.355	2.611	3.159	4.004	4.547	3.922	2.322	8.175	7.934	1.689
35D	3.466	5.122	4.447	4.605	3.602	3.369	4.260	4.772	3.922	2.322	8.175	7.934	1.689
44A	2.934	4.902	3.504	4.672	2.656	2.933	3.684	4.537	1.585	1.585	6.644	6.426	1.000
44B	3.256	4.126	3.147	4.126	2.851	2.703	2.781	4.209	1.585	1.585	6.644	6.426	1.000
44C	3.080	4.304	3.584	4.294	2.430	3.102	3.614	3.686	1.585	1.585	6.644	6.426	1.000
44D	3.158	4.072	3.126	4.209	3.209	2.778	2.697	4.310	1.585	1.585	6.644	6.426	1.000
59B	4.147	4.586	4.464	4.711	3.380	4.237	4.253	4.349	2.823	2.322	6.644	5.426	1.216
59C	4.314	4.685	4.555	4.490	4.260	3.264	4.120	4.355	2.722	2.322	6.644	6.426	1.172
59D	4.241	4.619	4.051	4.758	3.580	4.138	3.654	4.297	3.039	2.322	6.644	6.426	1.309
46	3.835	4.630	4.246	4.356	3.462	3.455	3.914	4.180	2.000	2.000	6.644	6.426	1.000
25	5.143	5.804	5.158	5.817	4.164	4.423	4.707	5.462			9.401	9.148	

Nevertheless, in terms of our rationale, it is possible to develop for these problems (36C, 36D and 25) uncertainty values under hypothesis H_1 and H_2. These are presented in Tables 21 and 22. The uncertainty values listed in column X depend exclusively on the minimum number of questions needed to solve the problem. For instance, in the case of Problem 31A, the tactics that will solve the problem are 6, 3, 10 and 6, 5, 9. As explained in CRP 1449 (Rimoldi, Fogliatto, Haley, Reyes, Erdmann & Zacharia) a table representing these sequences can be built. This table is used to score subjects' performance in terms of schemata norms. The uncertainty value of this table is 2.25163. Nevertheless, since tactics 6, 3, 10 and 6, 5, 9 are equivalent, and since the number of questions needed to solve the problem is 3 in both cases, the uncertainty value may be reduced to 1.58496, as given in column X in Tables 21 and 22. That is, the values in column "schema" and column "X" will be identical when there is one and only one selected tactic. The ratio between "schema" and "X" value will be 100 when these two are identical. The higher the value, the greater the uncertainty of the schema in relation to the uncertainty of one of the "ideal" performances. These values are given in the last column of Tables 21 and 22.

Analyzing reversal in uncertainty values for corresponding columns (groups of subjects) in Tables 21 and 22, it is found that in all the problems, the observed results show less uncertainty for the college group than for the high school group. These results are in all cases significant beyond the .01 level (one tailed sign test). The only exceptions being Groups III and IV. For group III, the sign test gives a significance of .039. For Group IV, no significance is found. These results confirm the expectation that college students, as a group, will perform with less uncertainty than high school students. With reference to Group IV, this statement has no statistical significance.

The comparisons that follow are based on the number of problems for which a given group presented higher or lower uncertainty values than another group. The test used has been again the sign test and the significances are for the one tailed test except when comparing Groups II and III. The reasons for this being that no clear directional hypothesis could be made in the comparison of these latter two groups. In Table 23, the number of problems for which uncertainty values are greater or smaller in one group than in another are given.

Comparison of Groups I and II with III and IV

These two groups represent high (I and II) and low academic grades (III and IV). In college, the high group has lower uncertainty values in a significantly larger number of problems. The significance is .001. The same results are found in the high school group with the significance being .026. This was to be expected in as much as there were significant differences between high and low groups for college and high school on the basis of the academic grade variable used in selection. Obviously, this indicates that group performances in these problems are sensitive to differences in academic achievement. The uncertainty values were lower in 21 of 22 problems for Group I and II in college and in 16 out of 22 for Group I and II in high school.

Comparisons of Groups I and III with II and IV

These two groups correspond to high (I and III) and low (II and IV) test scores. In both college and high school, the group with higher test scores has lower uncertainty values in a significantly greater number of problems. The significances are .002 and .001 for college and high school groups, respectively. This indicates that group performances in these problems relate to differences in performance in the tests used for selection. The uncertainty values were lower in 18 out of 22 problems for Groups I and III in college and in 21 out of 22 for Groups I and III in high school.

Comparison of Groups I, II, III and IV

The crucial comparisons refer to Groups I and IV, and II and III. Group I refers to subjects that were high in both selected criteria while Group IV refers to subjects low in both criteria. The number of problems in which the uncertainty value is less for Group I than for Group IV is: in college, 22 out of 22, and in high school, 18 out of 22. In both cases, these findings are highly significant as it was expected in terms of the design. In terms of the t values for tests, the differences are of about the same magnitude in both the college and high school groups. But in terms of the t values for grades, the difference is much greater for college than for high school. This may possibly influence the results as stated above where the discrimination in high school was not as good as in college.

The comparison of Groups II and III shows that the number of problems in which the uncertainty values is less for Group II than for Group III is: in college, 15 out of 21 (there is one tie) and in high school, 6 out of 22. Notice that the findings are in opposite directions though in both cases the significance does not reach the .05 level for a 2 factor test. The values are .078 and .052 for the college and high school groups, respectively.

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* The t values are as follows: high school differences between Groups I and IV, $t = 4.87$ $p < .002$, differences in Raven's and Thought Problems, respectively $t = 11.21$ $p < .0005$. For college: differences in grades $t = 8.76$ $p < .0005$, differences in Raven's test $t = 7.75$ $p < .0005$, differences in Thought Problems $t = 6.14$ $p < .0005$.

TABLE 23

(Sign Test)

Comparisons	High School	College
(I and II)<(III and IV)	16 out of 22,p=.026	21 out of 22,p<.001
(I and III)<(II and IV)	21 out of 22,p<.001	18 out of 22,p=.002
I<II	19 out of 22,p<.001	12 out of 21,p=.416
I<III	18 out of 22,p=.002	16 out of 22,p=.001
I<IV	18 out of 22,p=.002	22 out of 22,p<.001
II<III	6 out of 22,p=.052*	15 out of 21,p=.078*
II<IV	13 out of 22,p=.262	22 out of 22,p<.001
III<IV	18 out of 22,p=.002	22 out of 22,p<.001

* These are 2 tailed probabilities

Though no significant differences are found in the performance of Groups II and III in the experimental problems, the direction of the findings is consistent with the interpretation that Group II, being significantly superior in grades, should perform with less uncertainty than Group III in the college group. In high school, where Group III is less uncertain than Group II in terms of performance on the problems, grades are not significantly different, but tests show Group III to be significantly higher than Group II. Thus, though the results seem to be conflicting, the direction indicates that less uncertainty is associated with high scores in the criteria tests and/or academic grades.

This problem may have been clarified if Groups II and III were clearly differentiated in terms of academic grades and performances in the Raven's and Thought Problems tests; that is, if we had a significantly higher mean for grades in Group II than in Group III, and a significantly higher mean in test performance in Group III than in Group II.

Reference to Tables 17 and 18 indicate the above trend to hold for both groups, but the differences are greater for high school than for college in the two criteria tests. As a matter of fact, the difference in means for the Raven test in Groups II and III is significant at the .05 level in the hypothesized direction for the high school students and not for the college students. Concerning grades, a significant difference at the .001 level in the hypothesized direction is present between Groups II and III at the college level, but not at the high school level.

In order to interpret these findings, due consideration should be made of the fact that the multiple correlation between grades with the two criteria tests is .606 for the college group and .752 for the high school group. These significant correlations serve to explain the difficulty in achieving a clearer differentiation between Groups II and III.

The reader may examine Table 23 to study the remaining comparisons between groups.

The uncertainty values for all the problems in all the groups were plotted in graphs where the ordinate represents uncertainty values (Hxy) and the abscissa the specified Groups I, II, III, IV, (I and II), (III and IV), (I and III), and (II and IV). This ordering of the groups is arbitrary. On the same graphs are presented the maximum uncertainty according to H_1 and H_2 (referring to maximum uncertainty under hypotheses 1 and 2, respectively) and the minimum uncertainty as defined by the schema norms and by X as given in Tables 21 and 22. With the exception of Problems 35A, B, C and D in both high school and college students, the observed uncertainty values are above the uncertainty value for the corresponding schema. This could be explained considering that the definition of the schema norms is based on several tactics, each of which may be considered to be an ideal one. Therefore, the tables from which the uncertainty values are computed will show less agreement than if only one tactic were possible. This was to be expected considering the ratios between the schema and X columns in Tables 21 and 22. It is seen in these tables that this ratio is highest for Problems 35A, B, C and D and unity for those problems where the schema and the X values are identical, i.e., only one tactic is possible. Incidentally, this brings about the possibility of differentially weighting scores considering this discrepancy which is due to an artifact that is very difficult to control, in building and scoring the problems. This line was not pursued in this research. On the other hand, no group performances that are logically acceptable can ever fall below the X value corresponding to each problem. This was found to occur in Problem 42 for the high school Group I, where the uncertainty value is 1.97083 and the X value is 2.00000. Analyzing the performance of the subjects in this group, it was found that two of them attempted to solve the problem by asking these questions. By inspecting the problems as given in the Appendix, it can be seen that by asking questions 3, 1 and 5, in this order, as did these subjects, there is not enough information to assure the correct answer. The interesting fact is that one of these subjects gave the right answer while the other did not, in spite of the fact that both used the same tactic. It seems safe to conclude then that the correct solution was obtained by guessing, and this incidentally seems to be a good demonstration of our contention that correctness of the final answer does not vindicate the process. Therefore, the fact that the uncertainty value for Problem 42 in high school Group I falls below the X line does not invalidate our reasoning, but indicates that some subjects did not proceed in a logical manner throughout their performance.

Inspection of these graphs showed again that Group I shows less uncertainty than Group IV; similarly, Groups (I and II) and (I and III) show less uncertainty than Groups (III and IV) and (I and IV) respectively.

This can be observed by scanning Tables 21 and 22 or if desired, by considering the average of the uncertainty values for each group. Notice that the presently reported findings hold for high school and college students. Nevertheless, with reference to Groups II and III, the picture is not as clear and the average values indicate Group II to show less uncertainty than Group III in college while the opposite holds for high school. Of course, results of this kind should be expected on the basis of our previous discussion of the results obtained using the sign test.

It should be remembered that throughout the analysis of the uncertainty values in all the problems, no necessary connection is supplied between these uncertainty estimates and "goodness" of performance. Conceivably, a highly talented group could show a high uncertainty value because each member of this group may be able to discover a unique approach, all being equally excellent. Again, a very poor group may present a very low uncertainty value because the subjects will behave similarly in relation to the problem in spite of the fact that none of them proceeds logically. That is, consensus or disagreement are not necessarily a criterion of either "good" or "poor" performance. Nevertheless, it can be assumed that the probability of each of the above stated cases is typically low; that is, considering the nature of the problems and the selection of our group, these events are most unlikely to occur and a reasonable expectation will be that homogenous performance is likely to be related to "good" performance.

b) Pulling Out Method

The decision to use the pulling out method for scoring subjects' performances was based on previous experimentation and on the results obtained by Erdmann, 1964 (An appraisal of three scoring procedures as discriminators between good and poor problem solvers, Loyola Psychometric Lab, Pub. No. 40), where this approach was shown to be consistently better than other methods of scoring performance in the type of problems used in this research.

In essence, the pulling out method is used as follows: After determining by a logical analysis of the structure of the problem, the best tactic or tactics by means of which a problem can be solved, these are tabulated considering each question in each order in which it occurs in the various tactics, Rimoldi, Haley, Fogliatto and Erdman, (A program for the study of thinking, Bulletin, International Association of Applied Psychology, France, 1964 , and Loyola Psychometric Lab, Pub. No. 28, 1963). This table of frequencies is converted into a table of proportions. Each question will have a weight according to the frequency with which it occurs in a particular order. The next step is the application of this table of weights to an observed sequence. A previous method had merely summed the weights corresponding to the questions and the order of these questions in the observed sequence. This was called the schema method. The pulling out method "uses the same norms as the schema method and differs from it only in the application of the norms to the individual observed sequence. This method attempts to account for any restructuring or "late" understanding of the nature of the problem by the performer. In other words the benefit of the doubt is given to the subject in the evaluation of his performance.

"The procedure involves a kind of matching of the observed sequence with one of the ideal sequences. That is, the scorer determines the ideal sequence which best approximates the observed sequence and will therefore maximize the evaluation of the performer. Obviously, there are certain rules according to which this is done.

"The first step is to remove all the irrelevant (as far as the ideal sequence is concerned) questions from the observed sequence. It is important to maintain the order of the questions as selected by the subject.

"What results may be a complete or partial ideal sequence. In order to be complete, the order of the relevant observed questions must duplicate the ideal sequence. If this occurs, then one finds the value of the ideal sequence which would maximize the score for the observed sequence. This completes the second step in the determination of a final score for the pulling out method. The third and final step is to divide the value, found at the completion of the second step, by the number of questions of the original observed sequence, i.e., before any pulling out of irrelevant questions.

"The sequence resulting from the pulling out of irrelevant questions, however, may only partially duplicate an ideal sequence. In this case credit is given for the partial sequence. This value is again divided by the number of questions of the original observed sequence to determine the final score.

"An example of the technique is in order to clarify the application. Suppose the observed sequence 1,6,3,8,2,10. Assume that the ideal sequences of the problem are 6,3,10 and 10,3,6. Pulling out the irrelevant questions leaves 6,3,10 for the observed sequence. This exactly duplicates the ideal sequence 6,3,10 so the final score is the value of the 6,3,10 sequence in the schema norms divided by 6 (the number of questions from the original observed sequence). Had the original sequence been 1,10,8,3,2,6 then the ideal sequence 10,3,6 would have been duplicated with results exactly as above.

"In most instances, the ideal sequence will not be duplicated. Assuming the observed sequence 1,6,7,8,2,3,5, the ideal sequence approximating it best is 6,3,10. However, there is only partial approximation here, namely 6,3. The final score is, therefore, the value of 6,3 in the schema norms, divided by 7 (in this case). The remnants of the observed sequence following the pulling out of irrelevant questions must follow the order of one of the ideal sequences so that an observed sequence without 3 and 6 in it would obtain no value at all. If either occurred at the end of the sequence only that question would contribute any value. For instance, the observed sequence 1, 3, 8, 4, would have zero as a final score. The sequence 1, 3, 6, 5, 7 would have the value of 6 in the first position in the schema norms divided by 5.

"This technique, in summary, works to the advantage of the subject by giving him the benefit of the doubt as far as the occurrence of restructuring or reshaping the problem is concerned. It also incorporates the advantages of the schema method and adds the feature of differentially penalizing the subject for the prodigal selection of cards." (Erdmann, 1964, Loyola Psychometric Lab, Pub. No. 40.)

Graphs for each performance can be made by accumulating the corresponding values. In this section, we shall attempt to validate this type of score in terms of several independent criteria and to study how it helps in understanding processes.

Though the pulling out method eliminates irrelevant questions, these questions have had a position in the observed sequence. This is partly taken into accound by dividing the sum of weights by the "total" number of questions asked. It is possible then to consider how many questions were eliminated as well as questions that, because of their order of occurrence, received no value in spite of the fact that they are relevant in other positions. These questions will have a weight of zero, and in any method of graphing that will accumulate successive weights will not cause an ascending of the curve. Thus, plateaux of different lengths can be observed. If a subject has several of these, a score will be given to him representing the number of questions with zero weights that he used.

Each subject was scored in terms of the pulling out method and length of plateaux in all the problems for which schemas were developed.

The average pulling out scores were computed for each group and for each problem. Problems 25, 36C and 36D were not used in this part of the study because no schemata norms are available.

Scatter diagrams of pulling out scores with grades, as well as with scores in each of the criteria tests were made for the high school group and the college group. In all cases, linear relationships seem to hold. The pulling out scores used for each subject are based on the average of their scores in all the problems. In Table 24, the pulling out scores, as well as grades and scores in the criteria test are listed for all the subjects. In Table 25, the correlations between the four variables are given, for both the high school and college groups. All of them are significant at or beyond the .01 level with the exception of r_{24} in college that reaches a significance beyond the .05 level.

In general, the correlations are higher for the high school group than for the college group. Only r_{14} is significantly different (beyond the .05 level) when comparing high school with college students, the higher value corresponding to high school students. It is interesting that in terms of these findings, the variance in common between our problems scored using the pulling out method and Thought Problems is greater for the high school group than for the college group. If this is the case, then partialling out the scores in Thought Problems should produce greater changes in the correlations corresponding to the high school students than in those of the college students. Some of the partial correlations obtained are given in Table 26. Here are listed only those that we consider of interest in relation to our problem. The numbers in parentheses in the body of Table 26 correspond to percentage of variance in common explained by the corresponding correlation.

Inspecting the left half of Table 26, it is found that whenever variable 4 (Thought Problems) is partialled out, the corresponding correlation decreases considerably. Thus, the amount of common variance between pulling out and grades reduces from 55% to 5% when controlling for heterogeneity in Thought Problems. The correlation of pulling out scores with Raven's scores shows a decrement in common variance from 58% to 10%, when controlling for Thought Problems scores, and finally, the correlation of grades and Raven's shows a decrement in common variance from 34% to nothing under the same condition. That is, controlling for heterogeneity in Thought Problems scores decreases substantially the association that the pulling out method shows with the other variables. Similarly, the correlation between grades and Raven's scores is lowered when controlling for Thought Problems scores. Whatever function or functions estimated by

TABLE 24

Mean Pulling Out, Raven, Grade, and Thought Problem Scores for Each Subject in Each Group in High School and College

Subj.	Average SPO	Grades	Ravens	T.P.	Subj.	Average SPO	Grades	Ravens	T.P.
Group I	**HIGH SCHOOL**				**Group I**	**COLLEGE**			
01	.0568	92.25	50	19.00	01	.0521	3.47	60	19.00
02	.0411	88.75	59	16.00	02	.0514	3.65	59	18.00
03	.0444	77.00	56	15.50	03	.0658	3.33	58	18.00
04	.0539	96.00	56	19.00	04	.0429	3.28	58	17.50
05	.0444	92.25	58	15.50	05	.0346	3.00	58	19.00
Group II					**Group II**				
06	.0327	89.75	41	8.00	06	.0523	3.12	52	11.00
07	.0187	76.50	41	4.00	07	.0396	3.00	55	12.50
08	.0292	86.50	39	12.00	08	.0540	3.00	51	16.00
09	.0291	74.00	47	17.00	09	.0512	3.00	55	15.00
10	.0205	80.50	35	11.50	10	.0524	3.53	56	16.00
Group III					**Group III**				
11	.0199	83.00	50	8.50	11	.0456	1.94	54	14.50
12	.0317	76.75	41	12.50	12	.0435	1.29	53	15.50
13	.0361	76.50	51	14.00	13	.0348	1.41	53	17.00
14	.0261	69.00	48	10.00	14	.0414	1.47	57	16.00
15	.0221	82.75	50	11.50	15	.0544	1.65	57	16.00
Group IV					**Group IV**				
16	.0132	57.33	38	3.00	16	.0282	1.59	45	12.00
17	.0114	69.75	33	6.50	17	.0348	1.82	50	11.00
18	.0177	73.50	33	6.00	18	.0267	.94	47	12.00
19	.0188	61.50	36	5.50	19	.0127	1.71	51	16.00
20	.0219	72.67	36	5.50	20	.0241	1.88	51	12.50

TABLE 25

Correlations between Grades
Criteria Test and Scores in the Problems

	High School	College	Significance of Difference Using z Transformation
Pulling out - grades*, r_{12}	.74,p<.01	.61,p<.01	p=.4839
Pulling out - Raven, r_{13}	.76,p<.01	.60,p<.01	p=.3789
Pulling out - Thought Problems, r_{14}	.90,p<.01	.65,p<.01	p=.0424
Grades - Raven, r_{23}	.58,p<.01	.61,p<.01	p=.8966
Grades - Thought Problems, r_{24}	.75,p<.01	.45,p<.05	p=.1556
Raven - Thought Problems, r_{34}	.74,p<.01	.74,p<.01	

* The following code will be used:
Pulling out scores = 1
Grades = 2
Raven = 3
Thought Problems = 4

TABLE 26*

Correlations and Partial Correlations

High School					College				
Correlations	Variable Partialled out				Correlations	Variable Partialled out			
	1	2	3	4		1	2	3	4
r_{12}=.74 (.55)	-	-	.57, $p<.02$ (.32)	.23 (.05)	r_{12}=.61 (.37)	-	-	.38 (.14)	.47, $p<.05$ (.22)
r_{13}=.76 (.58)	-	.60, $p<.01$ (.36)	-	.32 (.10)	r_{13}=.60 (.36)	-	.36 (.13)	-	.23 (.05)
r_{14}=.90 (.81)	-	.78, $p<.001$ (.61)	.77, $p<.001$ (.59)	-	r_{14}=.65 (.42)	-	.53, $p<.02$ (.28)	.38 (.14)	-
r_{23}=.58 (.34)	-	-	-	.06 (.00+)	r_{23}=.61 (.37)	-	-	-	.46, $p<.05$ (.21)
r_{24}=.75 (.56)	-	-	.59, $p<.01$ (.35)	-	r_{24}=.45 (.20)	-	-	.00 (.00)	-

High School	College
$r_{12.34}$ = .22 (.05)	$r_{12.34}$ = .41 (.17)
$r_{14.23}$ = .67, p .01 (.45)	$r_{14.23}$ .41 (.17)

* 1 = Pulling out scores
2 = Grades
3 = Raven
4 = Thought Problems

Thought Problems may be, they influence grades and pulling out scores. This picture is further strengthened when $r_{12.34}$ is observed, that is, by controlling Raven's and Thought Problems scores. The amount of common variation between pulling out scores and grades reduces from 55% to 5%. On the other hand, when controlling for grades and Raven's scores $r_{14\cdot23}$, the common variance between pulling out scores and Thought Problems reduces from 81% to 45%, which is less than half with the correlation still being significant. Notice also that controlling grades or Raven's scores produces less obvious changes. That is, it seems as if in the high school group, the common variance corresponding to Thought Problems and to pulling out scores is substantiated. These two instruments seem to appraise functions that are important in determining school grades as seen from the relevant low partial correlations when the effects of Thought Problems are controlled. For the college group, the picture is slightly different. Partialling out grades produces greater changes than in the high school group, and partialling out Thought Problems scores produces smaller changes than in high school. It is difficult to interpret the effect of partialling out Raven's scores in the correlation between grades and Thought Problems, but $r_{12.34}$, when compared with the same index in high school shows less amount of change. On the other hand, controlling grades and Raven's scores ($r_{14\cdot23}$) the change is greater in college than in high school. This seems to indicate that, at least in our sample, test indicators are more closely related to estimates of ability in high school than in college, where grades seem to be better discriminators. Finally, it should be noticed that r_{14} in the high school group decreases slightly when controlling for grades on Raven's scores. In the college group, the effects are more noticeable.

These findings may be related to the manner of selecting our high school sample, but the picture above described agrees with the other findings of this study. Though it is true that the interpretation of a partial correlation coefficient is difficult, this can be done provided no unjustifiable assumption of cause is made. The findings have been analyzed here in terms of variance in common and the effects on this variance of the artificial control brought "a posteriori" by the partial correlation technique. An independent exploration would be needed to verify these results.

Comparisons between Groups (I and II) and (III and IV)

Group (I and II) corresponds to subjects with high academic grades. Group (III and IV) corresponds to subjects with low academic grades. The t-values for the differences in pulling out scores for all the problems are given in the second column of Tables 27 and 28. It should be noticed that there is a large number of significant values (using one tailed tests) indicating that the performance of Group (I and II) was higher than the performance of the low academic group in both high school and college students. In only one case,

TABLE 27

t-values of the Differences Between High (I & II) and Low (III & IV) Academic Groups and Between High (I & III) and Low (II & IV) Test Score Groups in the High School Based on the Pulling Out Method.

Problem	Group (I & II)-(III & IV)	Group (I & III)-(II & IV)
31 A	2.5071**	0.6696
31 B	2.2395**	1.4515
31 C	2.3209**	2.4052**
31 D	2.3123**	1.1987
40	1.9020*	0.7077
42	3.0106***	3.3493***
35 A	2.4713**	1.0315
35 B	2.1155**	3.4302***
35 C	0.9531	3.6124***
35 D	1.4492	2.9085***
44 A	3.3075***	2.3024***
44 B	1.5401	2.9459***
44 C	1.7602*	2.4714**
44 D	0.0827	1.6878
59 B	2.1129**	1.3437
59 C	1.5080	2.3916**
59 D	-1.2457	1.4006
46	2.7966**	2.4126**

* P<.05
** P<.025
*** P<.005

All Tests are One-Tailed

TABLE 28

t-values of the Differences Between High (I & II) and Low (III & IV) Academic Groups and Between High (I & III) and Low (II & IV) Test Score Groups in the College Based on the Pulling Out Method.

Problem	Group (I & II)-(III & IV)	Group (I & III)-(II & IV)
31 A	1.2489	1.3741
31 B	2.9877***	2.4183**
31 C	2.4237**	0.7574
31 D	2.3239**	0.7152
40	1.8987*	1.4066
42	0.9448	2.9695***
35 A	2.1362**	2.2739**
35 B	1.4422	2.0781*
35 C	3.0459***	0.4196
35 D	4.8620****	-0.2347
44 A	2.9746***	1.2467
44 B	0.3684	0.7609
44 C	1.9115*	0.2668
44 D	1.4870	-0.1900
59 B	1.9759*	1.2756
59 C	1.0974	1.4680
59 D	1.3514	2.4613**
46	1.8615*	-0.6332

* $P < .05$
** $P < .025$
*** $P < .005$
**** $P < .0005$

All Tests are One-Tailed

(Problem 59D, in high school) the difference is in the opposite direction though not significant at the prescribed levels.

Comparison between Groups (I and III) and (II and IV)

This amounts to establishing the sensitivity of our problems to variation in the criteria tests. As shown in the third column of Tables 27 and 28, the t-tests indicate that the differences are significant at the prescribed levels in twice the number of cases for the high school students. Also, in the college students, the differences in Problems 35D, 44D and 46 are in the opposite direction, though in no case do they reach a conventionally accepted level of significance.

The first conclusion seems to be that the problems used in this study are preferably related to grade averages in the college group. In the high school group the effects of grades and performance in the criteria test seem to be similarly effective in discriminating performances in these problems.

It would then be interesting to analyze the differences between the compared groups in the variables not used to define them. That is: Since Groups (I and II) and (III and IV) have been selected on the basis of academic grades, and are significantly different in terms of this criterion, then are Groups (I and II) and (III and IV) different in terms of the criteria tests (Raven and Thought Problems)? Similarly, are Groups (I and III) and (II and IV) different in terms of academic grades as well as in terms of the criteria used for their selection?

In Table 29, t-values are given for tests and grades in both high school and college students.

TABLE 29

t-Values and Probability Levels for Differences in Scores in Raven's Test, Thought Problems and Grades for Designated Groups

	Raven's Test	Thought Problems	Grades
High School Grades Group (I & II)(III & IV)	1.81,p<.10	2.25,p<.05	3.64*,p<.0025
High School Tests Group (I & III)(II & IV)	6.37*,p<.0005	4.99*,p<.0005	2.19,p<.05
College Grades Group (I & II)(III & IV)	2.67,p<.02	2.19,p<.05	13.44*,p<.0005
College Tests Group (I & III)(II & IV)	4.27*,p<.0005	4.44*,p<.0005	.22

* one tailed tests

It was thought that a one tailed test was indicated when selection was done on the variables being tested while a two tailed test was appropriate when the purpose was to discover whether groups defined in terms of one criterion were significantly different in terms of the other criterion.

As can be observed in Table 29, when the comparison is made in the vafiable or variables used for selecting the groups, the difference is always significant. These correspond to all the one tailed tests given in the table. However, when the comparison is made betweenn grades for groups selected in terms of the criteria test, it is found that in high school the difference is significant, t=2.19, but not in college, t=.22. And when Groups (I and II) and (III and IV) selected on the basis of grades are compared in terms of performance in the criteria test, the differences are significant in both tests for college, but only in one test for high school.

Thus, the following interpretation may be in order. Notice the third column in Table 27 which represents the differences in performance in the problems between Groups (I and III) and (II and IV) selected on the basis of the criteria tests. But these two groups are also significantly different in terms of grades as shown in the appropriate row of Table 29. This is interesting in view of the results found in the third column of Table 28, where the number of significant differences is half. But this refers to college students, where, as indicated in the last row of Table 29, there is no significant difference in grades. This combination of events seems to suggest that the additional significant differences in the third column of Table 27 may be due to the unique significant difference in grades for the high school group.

On the other hand, the events in column 2 of Tables 27 and 28 are almost parallel in terms of number of significant differences. Notice also that in Table 29 the t-values are similar for the two groups in question.

Comparison of Groups I, II, III and IV

In Tables 30 and 31, the columns refer to groups being compared and the entries are the corresponding t-values for each problem. We shall be concerned with the comparisons between Groups I and IV and II and III. For the same reasons given before in the case of the uncertainty values, a two tailed test will be used for the comparisons referring to Groups II and III, all the others justifying the use of one tailed tests.

It is quite clear that in terms of the performance in these problems, Groups I and IV are clearly differentiated. The differences are clearly in the hypothesized direction. The number of significant values at least at the $p<.05$ is 16 out of 18 in the high school group and 11 out of 18 in the college group.

TABLE 30

t-values of the Differences Between the Various Groups in the High School Based on the Pulling Out Method.

Problem	I-II	I-III	I-IV	II-III	II-IV	III-IV
31 A	1.3259	3.8897***	2.7304**	1.1359	.5809	-.9202
31 B	1.0412	.9034	4.5811***	.3916	3.2056**	1.2728
31 C	2.6687**	2.0936*	4.4341***	-.0511	1.6904	1.3125
31 C	1.6681	1.9409*	2.8662**	.6586	1.3094	.2804
40	.8862	1.5167	1.8918*	.7654	1.0839	.0076
42	10.1286****	8.3205****	14.4176****	-.3333	1.6321	1.7057
35 A	2.6002**	3.0046**	3.1311**	.8695	.7598	-.1536
35 B	5.2946****	2.7766**	5.007***	-.7932	1.4410	1.8928*
35 C	2.7922**	.9229	3.1791**	-1.7874	.7803	2.3014*
35 D	2.9445**	1.6018	6.6958****	.6973	.7546	1.4748
44 A	2.7685**	3.4088***	4.6285***	.8272	2.6707**	1.9429*
44 B	2.7411**	1.6758	4.1724***	-.7976	.8722	1.6624
44 C	2.4119**	2.7173**	3.3294**	-.4728	.4958	1.3019
44 D	.1869	-1.2841	1.3284	-1.0673	.8541	2.4649**
59 B	.6581	1.0493	2.3975**	.5230	2.4108**	1.6185
59 C	1.8623*	1.4816	3.3498**	-.4949	.8739	1.6838
59 D	1.1195	-.7727	.0558	-1.5530	-1.1084	.8968
46	3.7549***	3.3950***	5.6373****	.2472	1.5670	.8793

* P<.05
** P<.025
*** P<.005
**** P<.0005

TABLE 31

t-values of the Differences Between the Various Groups in the College Based on the Pulling Out Method.

Problem	I-II	I-III	I-IV	II-III	II-IV	III-IV
31 A	.4739	.4052	1.7411	-.0917	1.2745	1.3998
31 B	2.7157**	6.1715****	10.3435****	.2791	1.1851	2.2087*
31 C	1.1956	1.8812*	3.4077***	.8676	1.4485	.0658
31 D	.6066	2.0845*	1.7958	1.3143	1.3138	.4998
40	.8408	1.4754	2.8505**	.2652	1.3202	1.2583
42	1.2266	.0883	2.8187**	-1.3660	1.5465	3.4073***
35 A	1.0000	1.0000	3.0000**	-.2000	2.5733**	2.6983**
35 B	.7495	.4738	2.6575**	-.4437	1.5258	2.2502*
35 C	1.0000	2.2400*	2.6250**	1.5867	1.9007	.2020
35 D	-1.6842	3.1672**	2.6203**	4.2381***	3.3704***	.0585
44 A	-.2298	.8214	3.2494**	1.3110	4.8686***	2.4290**
44 B	-1.7036	-2.3973**	.8815	-.3906	2.3598**	2.9875**
44 C	.1695	1.0485	1.8387	.9303	1.7037	.2227
44 D	-2.1694*	.5087	.8593	1.6696	2.8924**	1.3461
59 B	.9693	1.4144	2.3994**	.4362	1.4042	.9772
59 C	.4021	.1416	1.6613	-.2838	1.6450	1.7050
59 D	2.3793**	.2202	3.1300**	-.6824	2.6300**	2.7800**
46	-1.7307	.0759	.9019	1.8364	4.5465***	.8069

* P<.05
** P<.025
*** P<.005
**** P<.0005

With reference to Groups II and III, there is only one significant difference in the college group and none for the high school group. Further, the direction of these differences indicates that in the high school group, 10 out of 18 favor Group III, while in college, the majority favor Group II. Since Group II refers to high grades and low scores in the criteria tests and since Group III refers to high scores in the criteria tests and low academic grades, then it can be advanced very tentatively that in college, grades are more closely related to good performance in the problems while in high school the criteria tests will show this preferential association. Again, the high correlation between grades and criteria test makes it difficult to state more definite conclusions.

Comparing Group I with II and III with IV, the effects of the criteria tests can be further appraised. For the first comparison, the number of significant differences is 11 out of 18 in high school and only 3 out of 18 in college. With reference to Group III with IV in high school, there are four significant differences and some of the remaining comparisons are in a direction opposite to that expected. In college, we find seven significant differences in the prescribed direction. In both instances, the assumption that Group III would be better than Group IV since Group III corresponds to subjects with high scores in the criteria tests, is supported.

These comparisons seem to indicate different pictures for those subjects with high academic grades (Groups I and II) and for those with low academic grades (Groups III and IV). Nevertheless, it should be considered that for the low academic group in high school, some of the problems may have been "too difficult" and this will reduce the number of significant differences.

Nevertheless, the criteria tests make a greater difference in high school than in college, and this is clearer for the high academic group than for the low academic group.

In the comparison of Groups I and III (both high in criteria tests) high school exhibits three significant differences more than college. However, in the comparison of Groups II with IV, (both low in criteria tests) college is observed to have the greater number of significant differences. Notice that in both these comparisons, any difference between the two groups in question should be seen in the light of the differences between the groups in the selection variables. In both comparisons for high school and for college, there are significant differences in academic grades. In the criteria tests there are significant differences between Groups I and III in high school and college and with these similar selection results high school seems better differentiated than college as indicated previously. In the criteria tests in the Group II with IV comparison, there is no significant difference in either high school or college. Considering these selection results, any difference should be due to the differences between the groups in academic grades, and consistent with previous findings, grades seem to discriminate better in college than in high school.

In summary, it seems that pulling out scores differentiates clearly between subjects at different academic levels and with different scores in the criteria tests. These differences seem to be related more heavily to differences in grades in college and to differences in criteria test scores in high school. It should be remembered that there is a high multiple correlation between grades and criteria tests scores in both of the groups. This together with the procedure used for selecting the high school sample, probably clouds a picture that shows the trend above indicated. But what remains clear is that pulling out scores are sensitive to changes in achievement and ability as expressed by grades and/or criteria tests.

Each subject's performance was also scored for each problem in terms of observed length of plateaux. Because of the previous related discussion, only some of the possible comparisons will be studied.

Tables 32 and 33 give the values of t for the length of plateaux when comparing Groups I and II against III and IV, and when comparing Groups I and III against II and IV. Table 32 refers to the high school group and Table 33 to the college group. The t-tests are one tailed in all cases because a direction can be safely assumed, favoring in each comparison the group that shows higher grades scores when comparing Groups I and II against III and IV, and favoring the group with higher scores in the criteria tests when comparing Groups I and III against II and IV.

Notice that longer plateaux indicate "worse" performance. Thus, the signs of the t-values should be negative.

Comparing Groups I and II against III and IV, as shown in the second column of Tables 32 and 33, it is found that 6 out of the 18 tests give significant differences in the high school group and 12 out of 18 in the college group. That is, the effects on performance in problems evaluated in terms of length of plateaux show that in high school the difference between the high and low academic groups (I and II against III and IV) is less noticeable than in college. On the contrary, when classifying subjects in terms of high (I and III) and low (II and IV) scores in criteria tests, it can be observed that the number of significant changes is greater for the high school group than for the college group. This trend verifies again our previous findings.

Analyses of variance were performed in terms of length of plateaux for problems 31 in all its forms, 35 in all its forms, 44 in all its forms and 59 in forms B, C and D. These analyses of

TABLE 32

t-values of the Differences Between High (I & II) and Low (III & IV) Academic Groups and Between High (I & III) and Low (II & IV) Test Score Groups in the High School Based on Plateaus.

Problem	(I & II)-(III & IV)	(I & III)-(II & IV)
31 A	-4.3578****	.1448
31 B	.1258	-1.4627
31 C	-1.0465	-2.7137**
31 D	-1.5378	-1.5378
40	-1.4958	-1.9702*
42	-2.0300*	-2.6881**
35 A	-1.4807	-1.1594
35 B	-.9865	-3.9290****
35 C	-.7390	-2.5476**
35 D	-.8633	-2.1069**
44 A	-3.1202***	-4.2893****
44 B	-2.0811*	-3.7331***
44 C	-2.2086**	-2.5017*
44 D	-1.3738	-2.2910**
59 B	-.8168	-1.0631
59 C	-.9185	-0.9186
59 D	.3974	-1.2362
46	-3.1462***	-2.7496**

* $p < .05$
** $p < .025$
*** $p < .005$
**** $p < .0005$

All are One-Tailed Tests

(Note on Direction of Sign: Negative Equals Better Performance.)

TABLE 33

t-values of the Differences Between High (I & II) and Low (III & IV) Academic Groups and Between High (I & III) and Low (II & IV) Test Score Groups in the College Based on Plateaus.

Problem	(I & II)-(III & IV)	(I & III)-(II & IV)
31 A	-1.3270	-.4243
31 B	-2.5726**	-2.1829**
31 C	-2.7088**	-1.1853
31 D	-2.8290**	.1385
40	-3.0704***	-.5452
42	-1.4831	-2.9104***
35 A	-1.8799*	-2.1927**
35 B	-1.2581	-1.9381*
35 C	-2.8222**	-.4729
35 D	-3.9384****	-.1605
44 A	-3.1332***	-1.8735*
44 B	-1.1748	-1.4100
44 C	-2.2700**	-0.9875
44 D	2.1685**	.4859
59 B	-2.3180**	-1.2744
59 C	-1.1931	-1.1931
59 D	-0.7478	-2.8988***
46	-2.3421**	-.2568

* P <.05
** P <.025
*** P <.005
**** P <.0005

(Note: Negative Sign = Smaller No. of Plateaus = Better Performance.)

All Tests are One-Tailed.

variance were performed independently for the high school and college groups. Because of the conditions of our design, the subjects belonging to Groups I, II, III and IV will be necessarily different. Thus, if we are interested in evaluating differences between groups as well as differences between contents (A, B, C and D) a pseudo three-way classification is indicated. This implies "nesting" of different groups of subjects in each of the "blocks" that in the case at hand correspond to Groups I, II, III and IV.

The results of these analyses of variance are given in Tables 34 and 35 for the high school and college groups respectively.

Notice that problem 59 behaves in very similar fashion in both the high school and the college groups. Nevertheless, the F test for I, II, III and IV shows a value of 3.219 which is almost significant at the 5% level. The value required being 3.240. If this is taken into account, then all the problems differentiate between Groups I, II, III and IV among college students, but only one, problem 44, in the high school group. Problem 35 almost reaches the 5% level of significance. In terms of content, only problem 31 shows a significant F value in both the high school and college group, while in problem 44, this only occurs in the high school group. Judging by these results, it seems as if the problems have greater differentiating possibilities in the college group than in the high school group. This may be partially related to the characteristics of our sampling procedure for the high school group and to the fact that some of the problems may have been too difficult for the high school students. This, in all likelihood, would tend to make their performance more similar.

With regard to the problems, it may be concluded that when scored in terms of length of plateaux, some of the problems built around schema 31, 35, 44 and 59 are able to differentiate between our subsamples of college students. For the high school group, this oc occurs for problems with schemata 44 and possibly 35 (see Table 34).

It would be risky to go beyond these general statements on the basis of the results provided by the analyses of variance, but these findings together with those presented before indicate that the study of processes as pursued in this research is not only a valid procedure, but also one which leans to a sensitive characterization of those processes.

c) Information Curves

In this part of the study, we shall present a method of scoring individual performances in terms of the processes followed by a subject in solving a problem. This method is independent of group performances and attempts to analyze at each step in the process the amount of uncertainty reduction with respect to the total uncertainty

TABLE 34

Analysis of Variance for Problems of Series 31, 35, 44 and 59 of the High School Students Based on Plateaus

Source	Σ of Square	df	Variance Estimate	F	Sig.
31					
Individuals	95.40	16	5.962		
I,II,III,IV	46.05	3	15.350	2.575	
A,B,C,D	16.25	3	5.417	4.907	1%
Interaction	23.25	9	2.583	2.340	5%
Remainder	53.00	48	1.104		
Total	233.95	79			
35					
Individuals	305.90	16	19.119		
I,II,III,IV	183.85	3	61.283	3.205*	
A,B,C,D	7.25	3	2.417	.860	
Interaction	20.85	9	2.317	.824	
Remainder	134.90	48	2.810		
Total	652.75	79			
44					
Individuals	53.600	16	3.350		
I,II,III,IV	207.138	3	69.046	20.611	1%
A,B,C,D	30.738	3	10.246	7.150	1%
Interaction	22,212	9	2.468	1.722	
Remainder	68.800	48	1.433		
Total	382.488	79			
59					
Individuals	104.267	16	6.517		
I,II,III,IV	13,667	3	4.556	.700	
B,C,D	.100	2	.050	.043	
Interaction	5.233	6	.872	.747	
Remainder	37.333	32	1.167		
Total	160.600	59			

* 3.240 is Significant at 5% Level.

TABLE 35

Analysis of Variance for Problems of Series 31, 35, 44, and 59 of the College Students Based on Plateaus

	Source	Σ of Square	df	Variance Estimate	F	Sig
#31	Individuals	29.700	16	1.856		
	I,II,III,IV	46.600	3	15.533	8.369	1%
	A,B,C,D	24.700	3	8.233	5.526	1%
	Interaction	17.300	9	1.922	1.290	
	Remainder	71.500	48	1.490		
	Total	189.800	79			
#35	Individuals	94.800	16	5.900		
	I,II,III,IV	121.500	3	40.500	6.900	1%
	A,B,C,D	14.500	3	4.800	1.800	
	Interaction	33.600	9	3.700	1.400	
	Remainder	128.400	48	2.700		
	Total	392.800				
#44	Individuals	56.900	16	5.790		
	I,II,III,IV	66.738	3	38.150	6.589	1%
	A,B,C,D,	2.534	3	.850	.659	
	Interaction	64.312	9	1.840	1.426	
	Remainder	97.700	48	1.290		
	Total	288.388				
#59	Individuals	118.267	16	7.392		
	I,II,III,IV	71.383	3	23.793	3.219*	
	B,C,D	1.233	2	.615	.282	
	Interaction	16.367	6	2.728	1.252	
	Remainder	69.733	32	2.179		
	Total	276.983				

* 3.240 is significant at 5% level

of the problem. This was accomplished for those problems in which figures are presented. (This method could be extended to problems of type 31, 35 and 59 where though no figures are given, there is still a schema that will permit, for every question asked an evaluation of the possibilities that are eliminated by the answer to this question.)

We are assuming that at the outset of the problem, all the alternatives are equally likely. On this basis the total uncertainty value for each problem can be calculated according to the total number of these equally likely alternatives. Then in order to solve the problem in a logical fashion or at least in a fashion eliminating guessing or hunches it becomes necessary for a subject to reduce the uncertainty of a problem to zero. This he can do by asking questions and obtaining the corresponding answers. Each of these questions and answers will reduce the total uncertainty according to the information contained in them and the order in which they were asked. It thus becomes a matter of determining for each question and answer that part of the total uncertainty of the problem that is eliminated according to the order in which it was asked. The next step is to consider an individual sequence of questions and to establish the amount of uncertainty reduction accomplished by each in the particular order it was asked. These values can be accumulated for individual questions of an observed sequence in the order in which they were asked so that at any point in the process, the amount of uncertainty reduced can be indicated. Obviously, certain questions asked in a definite order can maximize this process of uncertainty reduction, and this can be seen more clearly by plotting the cumulative values for successive questions. Tactics following the "best" sequence(s) as determined by the schema should be those that maximize the reduction of uncertainty with each successive question.

According to the nature of the problems, the solution to be obtained will be achieved through a process of either rejecting or retaining certain alternatives. This binary characteristic of the process suggests the use of a system of evaluation which embodies these properties, such as the transformations implied in "information theory".

The procedure can be illustrated by means of two examples: The first corresponds to a subject who followed the "best" sequence and the second to a subject who followed a "poor" sequence. In both cases, problem 42 is to be used.

The best sequence of questions for this problem is as follow: 3, 1, 5 and 8. The total number of alternatives contained in the problem is 24 which gives a total uncertainty value of 4.58496 bits. After asking question 3, the subject eliminates 12 areas, with 12 remaining for consideration, that is, there remain 3.58496 bits of uncertainty, so that question three reduced the uncertainty by one bit. After asking question 1, out of the 12 remaining alternatives 6 are eliminated, which leaves 6 alternatives with an uncertainty of 2.58496 bits. That is, question 1 in the second order has an uncertainty reduction value of 1.00000 bit. Therefore, with questions 3 and 1 in this order, the subject has reduced his uncertainty by 2 bits, so that the remaining uncertainty at this point will be 4.58496-2.00000=2.58496 bits.

Question 5 will eliminate 4 out of the 6 remaining alternatives This represents a remaining uncertainty of 1 bit and the uncertainty reduction value of the question will be 2.58496-1.00000=1.58496. Thus, questions 3, 1 and 5, in this order, have reduced the uncertainty by 3.58496 bits. Finally, selecting question 8 eliminates one alternative thereby yielding the solution since out of the two remaining alternatives, identification of one means the identification of both. The uncertainty value was one bit prior to asking question 8, and since in effect two alternatives were eliminated, this question has an uncertainty reduction value of one bit; hence, no uncertainty is left. This can be clearly seen by inspecting Table 36.

TABLE 36

(Problem 42 - Table 3158)

Questions	Remaining Alternatives	Uncertainty Reduction Value of Question	Cumulative Uncertainty Reduction
-	24	-	-
3	12	1.00000	1.00000
1	6	1.00000	2.00000
5	2	1.58496	3.58496
8	1	1.00000	4.58496

The values of the last column of Table 36 can be graphed to show uncertainty reduction at every step in the process.

In Table 37 for the same problem, we present the uncertainty reduction value of each question for the sequence 10,1,3,5,4,2.

TABLE 37

(Problem 42. Tactic 10, 1, 3, 5, 4, 2)

Questions	Remaining Alternatives	Uncertainty Reduction Value of Question	Cummulative Uncertainty Reduction
-	24	-	-
10	23	.06140	.06140
1	18	.35363	.41503
3	6	1.58496	1.99999
5	2	1.58496	3.58495
4	2	.00000	3.58495
2	2	.00000	3.58495

Notice that tactic 10, 1, 3, 5, 4, 2 has an uncertainty reduction value of 3.58495, while the problem has a total uncertainty of 4.58496 bits. Notice further that the same question, for instance 1, asked in both tactics in second order has an uncertainty reduction value of 1.0000 when preceded by 3 and an uncertainty reduction value of 1.58496 when preceded by 10. Similarly, question 5, which in tactic 10, 1, 3, 5, 4, 2 has an uncertainty reduction value of 1.58496 in tactic 3, 1, 5, 8 also has an uncertainty reduction value of 1.58496 bits, despite the fact that this question occurs in different orders.

This procedure can be used to evaluate each individual's performance in each problem independently of group performances. It is a method based on the logical properties of the problem and takes into consideration the order of the questions and their content.

The following graphs correspond to performance scores using the approach described above. For problem 42 in the high school group, the results are presented in Figure 59. The "best" tactic, according to the schema of the problem is 3, 1, 5, 8. The subjects of Group I used the following tactics:

3, 1, 5 (Subjects 1 and 3)
3, 1, 5, 8 (Subjects 2, 4 and 5)

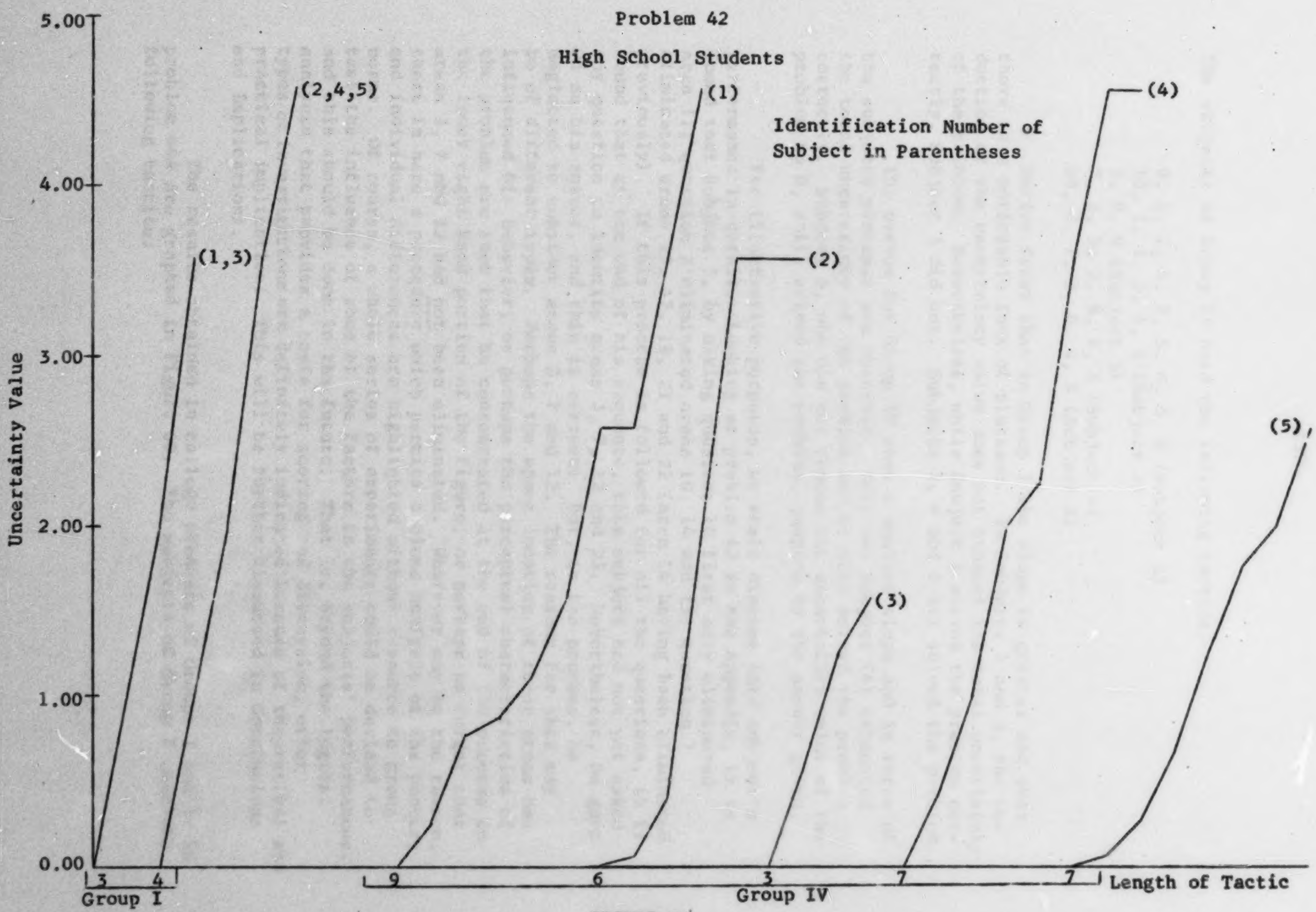
Problem 42
High School Students
Identification Number of Subject in Parentheses
Uncertainty Value
5.00
4.00
3.00
2.00
1.00
0.00
(1,3)
(2,4,5)
(1)
(2)
(3)
(4)
(5)
3
4
Group I
9
6
3
7
7
Group IV
Length of Tactic

FIGURE 59

The subjects of Group IV used the following tactics:

9, 1, 2, 4, 7, 3, 6, 5, 8 (Subject 1)
10, 1, 3, 5, 4, 2 (Subject 2)
5, 8, 9 (Subject 3)
7, 1, 3, 2, 5, 8, 6 (Subject 4)
10, 2, 7, 3, 8, 4, 5 (Subject 5)

Notice first that in Group I the slope is greater and that there is a noticeable lack of plateaux. In Subjects 1 and 3, the reduction of the uncertainty value does not exhaust the total uncertainty of the problem. Nevertheless, while Subject 1 solved the problem correctly, Subject 3 did not. Subjects 2, 4 and 5 all solved the problem..

The curves for Group IV show a smaller slope and in three of the subjects plateaux are observed. Only one subject (4) exhausted the total uncertainty of the problem and he also solved the problem correctly. Subject 5, who did not reduce the uncertainty value of the problem to 0, still solved the problem, judging by the answer given.

For illustrative purposes, we shall discuss this subject's performance in detail. Looking at problem 42 in the Appendix, it is found that Subject 5, by asking question 10 first only eliminated area 11; question 2 eliminated areas 10, 14 and 15; question 7 eliminated areas 13, 17, 18, 21 and 22 (area 14 having been eliminated previously). If this process is followed for all the questions, it is found that at the end of his sequence, this subject had not yet asked any question to identify areas 3, 7, 12 and 23. Nevertheless, he gave 23 as his answer, and this is correct. But, in the process, he neglected to consider areas 3, 7 and 12. The reasons for this may be of different types. Perhaps the space location of these areas has influenced his behavior; or perhaps the perceptual characteristics of the problem are such that he concentrated at the end of the process on the lower right hand portion of the figure; or perhaps he forgot that areas 3, 7 and 12 had not been eliminated. Whatever may be the reason, there is here a procedure which permits a close analysis of the process and individual differences are highlighted without resource to group norms. Of course, a whole series of experiments could be devised to test the influence of some of the factors in the subjects' performances, and this should be done in the future. That is, beyond the logical analysis that provides a basis for scoring and discussion, other types of investigations are definitely indicated because of theoretical and practical implication. This will be further discussed in Conclusions and Implications.

The results obtained in college students of Groups I and IV in problem 44A are graphed in Figure 60. The subjects of Group I used the following tactics:

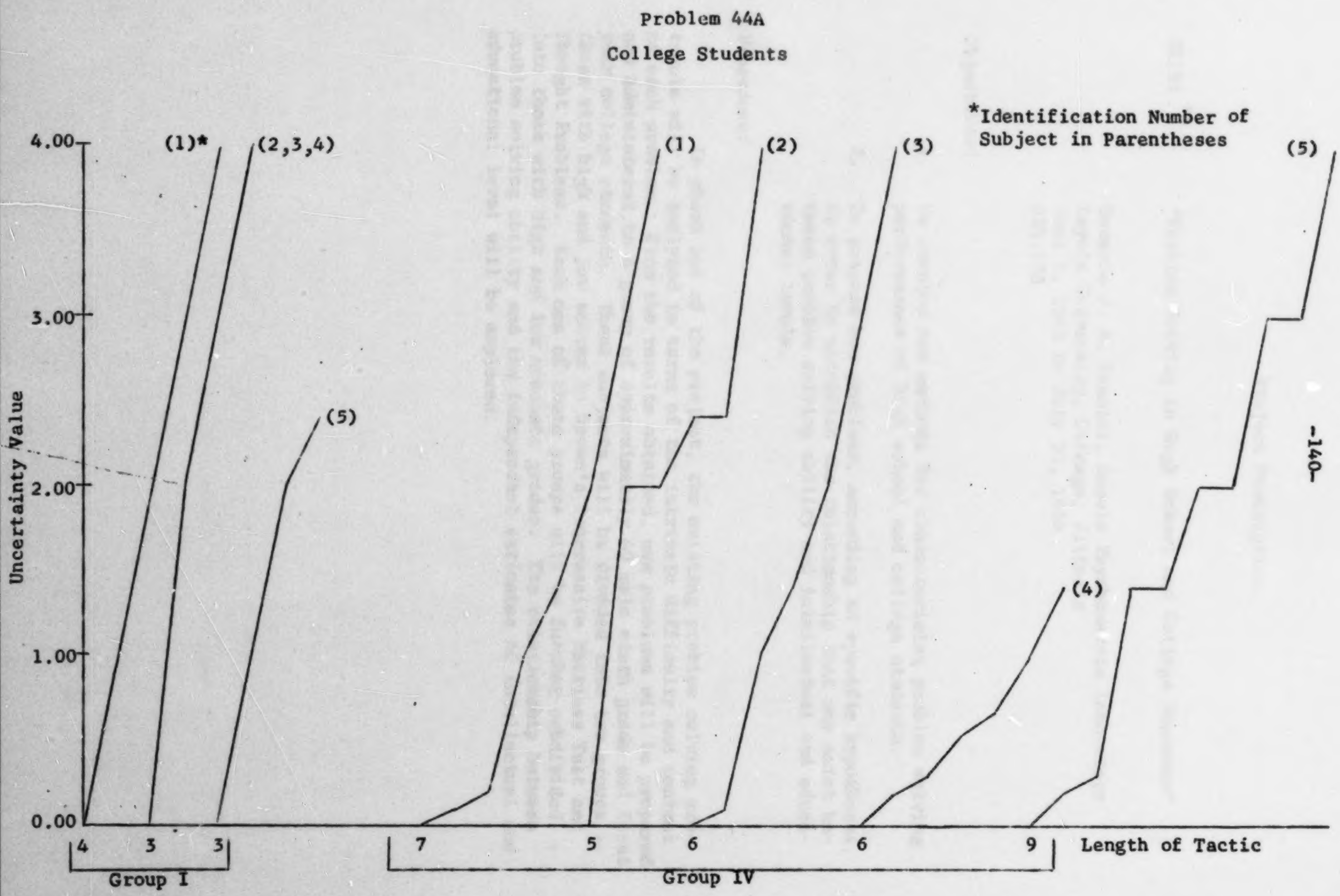
Problem 44A
College Students
*Identification Number of Subject in Parentheses
Uncertainty Value
4.00
3.00
2.00
1.00
0.00
(1)*
(2,3,4)
(5)
(1)
(2)
(3)
(4)
(5)
4
3
3
7
5
6
6
9
Length of Tactic
Group I
Group IV

FIGURE 60

Project Description

#2199 "Problem Solving in High School and College Students"

Horacio J. A. Rimoldi, Loyola Psychometric Laboratory
Loyola University, Chicago, Illinois
June 1, 1963 to July 31, 1964
$23,170

Objectives:

1. To develop new methods for characterizing problem solving performances of high school and college students.
2. To prepare new problems, according to specific hypotheses, in order to establish the relationship that may exist between problem solving ability and intellectual and educational levels.

Procedure:

In phase one of the project, the existing problem solving protocols will be analyzed in terms of the intrinsic difficulty and content of each problem. From the results obtained, new problems will be prepared and administered to a group of approximately 40 male ninth grade and first year college students. These subjects will be divided into two groups -- those with high and low scores in Raven's Progressive Matrices Test and Thought Problems. Each one of these groups will be further subdivided into those with high and low academic grades. The relationship between problem solving ability and the independent estimates of intellectual and educational level will be explored.

7, 5, 9 (Subjects 2, 3 and 4)
3, 7, 5, 9 (Subject 1)
3, 7, 9 (Subject 5)

The best sequence for this problem is 7, 5, 9, the total uncertainty being 4.0000 bits. All the subjects in Group I gave the "correct" answer to the problem, though Subject 5 did not reduce the total uncertainty of the problem to zero by his questions. In essence, this means that Subject 5 has not eliminated all the alternatives and thus his answer may be considered a "lucky guess". With reference to Figure 60, see that these subjects show consistently rapidly ascending uncertainty reduction curves.

Among the subjects in Group IV, the following sequences were observed:

2, 6, 3, 1, 7, 5, 9 (Subject 1)
7, 3, 9, 1, 5 (Subject 2)
6, 3, 1, 7, 5, 9 (Subject 3)
1, 2, 5, 6, 9, 10 (Subject 4)
1, 2, 3, 4, 5, 6, 9, 10, 7 (Subject 5)

With the exception of Subject 4, the rest reduced the uncertainty value of the problem to zero. All the subjects in this group solved the problem correctly including Subject 4. Nevertheless, Subject 4 failed to consider as possible answers a good number of areas. Subject 5 besides poor planning, judging by the small number of areas eliminated in early stages of the process, shows a considerable amount of redundancy. For instance, inspecting his sequences in relation to the problem, it is found that some of the areas were eliminated up to those times by the successive questions while, of course, only once would have been sufficient.

From these and other similar observations, it seems possible that neither correct answers nor economy in the number of questions asked and/or their combination are valid appraisers of processes.

In previous studies, training in Problem Solving, CRP 1449, a detailed description and discussion of these measures was given. In the present study we shall only present tables showing the number of correct solutions for high school and college students in the various groups. These tables are given in Appendix II. The picture is much the same as that which has repeatedly occurred in the analyses that have preceded. Nevertheless, it is well to point out again that the characteristics of the processes are not well represented by these values.

CHAPTER VIII

Conclusions and Implications

The results obtained in Phase I have already been discussed and, in this chapter, we will incorporate them as seems appropriate.

The basic problem was the investigation of problem solving with special attention given to intellectual ability, academic grades and educational levels as variables which may affect the problem solving situation.

Intellectual ability was appraised by means of the Raven's Progressive Matrices Test and by Thought Problems by Tate Stanier and Harootunian. Academic grades are based on averages of scholastic achievement as given by the schools. Educational level refers to high school and college groups.

Problem solving was investigated using some problems already prepared and found to be effective in CRP 1449. On the basis of this previous experience and of the results obtained in Phase I, eleven new problems were devised.

One of the major aspects of research of this nature concerns the development of effective techniques for evaluation. In the course of working with instruments such as these, various techniques were applied through the years, each with varying degrees of success. In this project some of the old approaches based on utility indexes, group performances, number of correct solutions, number of questions used, ellipsoids, etc., have been relegated to minor importance because of their qualified effectiveness in evaluating performance. Nevertheless, these other approaches should not be completely neglected because each one of them offers,information of a limited kind and are appropriate for very specific purposes; for instance, the use of group norms and ellipsoids to evaluate agreement of subjects with a normative group, or to evaluate agreement among the subjects of the group.

At the present stage of our investigations, it was necessary to attempt to devise scoring procedures that would allow the characterization of processes at an individual level, on the basis of an objective criterion. This was clearly indicated in previous studies (CRP 1449) that there were shortcomings in evaluating subjects by using group norms or by referring exclusively to the final answers given to the problems. In a way, these approaches ignored some valuable information and were not entirely consistent with our aims.

The concept of the logical structure of a problem and its use for developing appropriate schemata norms was a major impetus in the evaluation of individual tactics (sequence of questions). In the present report, a comparative study was performed in which the relative merits of several of these approaches were analyzed. These results are given in Appendix III. The result was that the so called "pulling out" method was the most appropriate technique to evaluate tactics. These conclusions, of course, are based on a limited sample of problems and subjects and did not consider techniques that we have since explored, such as the analysis of uncertainty reduction during the process.

In reference to this latter approach several points should be considered. In Chapter VII a detailed description is presented. It is interesting that for each problem, maximum and minimum uncertainty limits can be established without reference to any observed performance. The definition of maximum and minimum uncertainty is closely dependent on the characteristics of the problems as defined by the logical relationships inherent in each. With regard to the minimum uncertainty limit for a given problem, two different estimations can be made. These have been described in the text but the important connotation is that the ratio between "schema" and X values allows a characterization of the experimenter's success in building problems. Notice that this is possible independently of any subject's performance and provides an intriguing way of controlling the experimenter's instruments and his ability to build clear problems. That is, problems can be partially characterized in an objective manner prior to their administration. The experimenter's error, at least in this limited area, can be controlled unless, of course, problems with known ambiguity are desired to explore specific points.

The concept of uncertainty can also be used to evaluate performances of a group on the assumption that agreement in tactics may distinguish groups at different levels of ability and academic achievement. An explanation of this assumption is contained in the text (Chapter VII, a). This approach is also new to this research.

Both the pulling out method and the method of uncertainty reduction facilitate the analysis of the dynamic aspects of thinking which are ignored when the final answer is considered alone. As shown in the examples cited in the text, these methods make it possible for the experimenter to make definite hypotheses regarding hunches that each subject may make and other aspects of the problems that are either ignored or highlighted by the subject. This has great possibilities for research on thinking processes based on actual observations. As a matter of fact, the problems of originality and creativity may be advantageously explored by these means. That is, a logical frame as used in the building and scoring of problems provides perhaps the easiest frame of reference that the experimenter may use to interpret problem solving performance, but this need not be the only or the "best" characterization possible. As a matter of fact, it is from this fact that a whole plan of research could be initiated, pivoting oh well controlled instruments. The analysis of uncertainty reduction offers the possibility of isolating the contribution of various aspects of the problem to the process of solution. For instance, for problems with an identical logical structure variations in content, perceptual characteristics, etc., will produce changes in the uncertainty reduction value of various questions in different subjects. Thus, it may be possible to predict, by operating experimentally with these features of the problems in a planned manner, how each individual's process of thinking is affected. Of course, we are assuming that the subject is able to handle the logical structure of the problem and related research is now being conducted to explore this point. It is quite obvious that the educational implications of this are very direct, inasmuch as the emphasis on prescribed best approaches may hinder or stifle originality.

The study of individual tactics and their classification into possible families has been explored in Phase I of this study. The general implications are presented in the corresponding text. Similarly, the study of plateaux in the performance curves has been detailed in Phase I of this report as well as in Phase II. In general, these plateaux reflect the non-adherence to the "best" sequences determined by the logical structure.

In both Phase I and Phase II it has been found that male college freshmen perform better than male high school freshmen in these problems. In Phase I the effects of training have been analyzed and the major findings are given in the corresponding text, indicating that, on the whole, the experimental subjects perform better than the control subjects (those without training).

The pulling out scores show in all cases a high positive correlation with both grades and criteria tests, (Table 25). Only one significant difference is found between these correlations for the high school and college groups. This refers to the relationship between pulling out scores and Thought Problems. Partialling out grades and criteria tests (Table 26), it is found that at the high school level the criteria tests have greater influence than at the college level, where partialling out grades has greater importance. This may be due in part to the selection procedure used to identify the various groups in the design. In terms of grades and criteria tests, Groups I and IV are clearly identified while II and III show a considerable overlap. The possibility of obtaining groups more sharply delineated by the selection variables seemed to be a rather difficult, if not impossible task when considering the multiple correlation between grades and the criteria tests.

With reference to the validity of the pulling out method as a technique for evaluating processes, it may be concluded that the problems used here give essentially the same picture as grades and/or criteria tests. The finer difference between the groups at the high school and college levels in terms of selection variables are reflected in an almost parallel manner by the scores of the pulling out method. Notice that the above evaluation of a subject's problem solving ability is performed in terms of his tactic rather than in terms of the correctness of the final answer. Thus, this procedure allows one to be able to distinguish between processes, though the results of all of these processes may have been identical. Consequently, the correctness or wrongness of an answer seems to omit vital information. Typically, a subject who follows the "best" tactic has a great probability of reaching a correct solution, but correct solutions may also be reached by using tactics that deviate considerably from the one expected in terms of the logical structure of the problem. As it has been shown in the text, a subject may reach the correct answer without reducing his uncertainty to zero. Thus, while criteria tests and/or academic grades may differentiate between good and poor problem solvers, this is done mainly on the basis of the analysis of the answers and there is no ready way of characterizing the individual process involved in an objective manner.

Essentially, the same results are forthcoming from the analysis of the plateaux, of the group uncertainty values, and of the information curves. In summary: the development of new problems and of new methods of scoring problem solving performances has been successful. This opinion is based on the accurate differentiation of the groups that were selected in terms of grades and criteria tests, by the various types of scores and performance curves used in this study. Further, the problems and methods used allow one to follow closely the tactic of the subject and to obtain meaningful information about his process concerning possible hunches, redundancy, blind alleys, insufficient verification of clues, etc.

Many of the conclusions stated in CRP 1449 seem to be strengthened in this research. Besides the purely psychological connotations that our findings may have, a procedure has been presented that allows educators to prepare instruments specially designed to study the thinking processes of students, and to improve their own approach to teaching methods. Clearly, the emphasis has been on individual performances, and this makes this method a sensitive technique to select high talent, originality and creative ability.

This type of study should be pursued in the future. As indicated in the previous text, a planned approach can be made in which definite clues with clear educational connotations may be used. This possibility is now much closer to realization than at the earlier stages of our research.

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Problem 25

Instructions and Corresponding Questions and Answers

The figure here is composed of overlapping geometric figures and lines, forming twenty-five areas. You will notice that an area is any enclosed part of the figure that does not have a line through it. One of the areas has been selected. Your task is to find out which one it was. To find the particular area, you must discover enough facts about it so that it can be distinguished from other areas. You may discover these facts by using any of the questions you like.

Proceed by reading over all the questions. Then decide on the first question you would like to have answered, and write its number on the page provided. Then take the card and read the answer on the back of the card. After having read the answer, decide on the next question you would like to have answered. Write down its number, and take the card. When you are satisfied that you have arrived at the answer, stop drawing cards, and write down your answer. Remember. you may use as many of the cards as you need to find the correct area, but don't draw any more than you need.

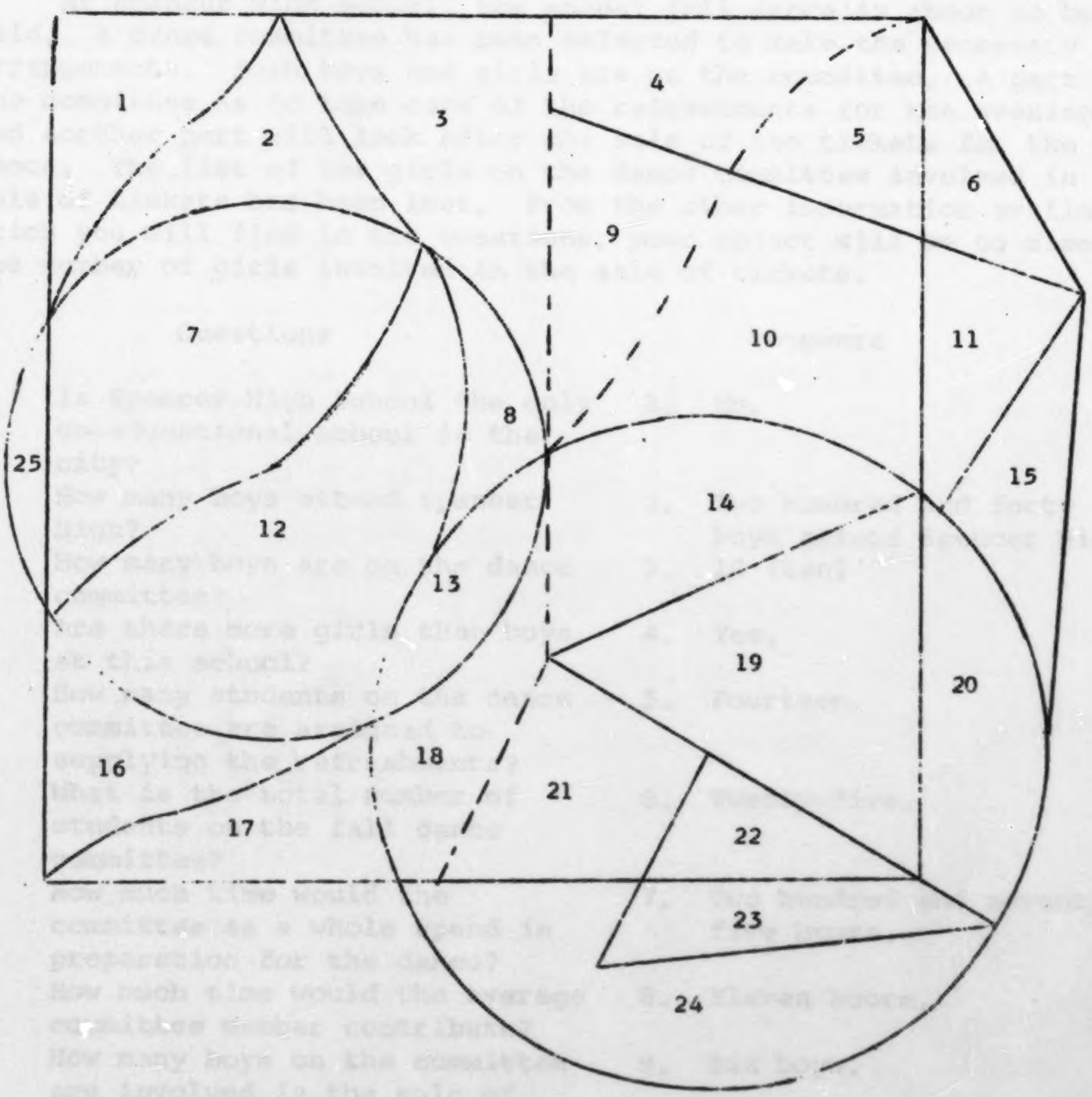
	Questions		Answers
1.	Is it in the large square?	1.	Yes, it is in the large square.
2.	Is it in the large circle?	2.	No, it is not in the large circle.
3.	Is it in the small circle?	3.	No, it is not in the small circle.
4.	Does the area have at least one right angle?	4.	No, it does not have at least one right angle.
5.	It the area a rectangle?	5.	No, the area is not a rectangle.
6.	Is the area a triangle?	6.	No, the area is not a triangle.
7.	Is it outside the large square?	7.	No, it is not outside the large square.
8.	Is it outside the large circle?	8.	Yes, it is outside the large circle.
9.	Is it to the left of the straight dotted line?	9.	No, it is not to the left of the straight dotted line.
10.	Is it to the right of the straight dotted line?	10.	Yes, it is to the straight dotted line.

Problem 25
(Continued)

	Questions		Answers
11.	Does it have all straight line boundaries?	11.	No, it does not have all straight line boundaries.
12.	Does it have all curved boundaries?	12.	No, it does not have all curved boundaries.
13.	Does the area have at least two boundaries?	13.	Yes, the area does have at lease two boundaries.
14.	Does the area have at least three boundaries?	14.	Yes, the area does have at least three boundaries.
15.	Does the area have at least four boundaries?	15.	Yes, the area does have at least four boundaries.
16.	Does the area have at least five boundaries?	16.	No, the area does not have at least five boundaries.
17.	Does it have at least two straight line boundaries?	17.	Yes, it has at least two straight line boundaries.
18.	Does it have at least three straight line boundaries?	18.	No, it does not have at least three straight line boundaries.
19.	Does it have at least four straight line boundaries?	19.	No, it does not have at least four straight line boundaries.
20.	Does it have at least two curved boundaries?	20.	Yes, it does have at least two curved boundaries.
21.	Does it have at least three curved boundaries?	21.	No, it does not have at least three curved boundaries.
22.	Does it have at least four curved boundaries?	22.	No, it does not have at least four curved boundaries.
23.	Does it have a dotted boundary?	23.	Yes, it does have a dotted boundary.
24.	Does it have two dotted boundaries?	24.	No, it does not have two dotted boundaries.
25.	Does it have a straight line boundary?	25.	Yes, it does have a straight line boundary.
26.	Does it have a curved boundary?	26.	Yes, it does have a curved boundary.

Solution: 10

PROBLEM 25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Appendix I

Problem 31 A

Instructions and Corresponding Questions and Answers

At Spencer High School, the annual fall dance is about to be held. A dance committee has been selected to make the necessary arrangements. Both boys and girls are on the committee. A part of the committee is to take care of the refreshments for the evening and another part will look after the sale of the tickets for the dance. The list of the girls on the dance committee involved in the sale of tickets has been lost. From the other information available, which you will find in the questions, your object will be to discover the number of girls involved in the sale of tickets.

Questions	Answers
1. Is Spencer High School the only co-educational school in the city?	1. No.
2. How many boys attend Spencer High?	2. Two hundred and forty boys attend Spencer High.
3. How many boys are on the dance committee?	3. 10 (ten)
4. Are there more girls than boys at this school?	4. Yes.
5. How many students on the dance committee are assigned to supplying the refreshments?	5. Fourteen.
6. What is the total number of students on the fall dance committee?	6. Twenty-five.
7. How much time would the committee as a whole spend in preparation for the dance?	7. Two hundred and seventy-five hours.
8. How much time would the average committee member contribute?	8. Eleven hours.
9. How many boys on the committee are involved in the sale of tickets?	9. Six boys.
10. How many girls are on the refreshment part of the dance committee?	10. Ten girls.

Solution: 5

Problem 31 B

Instructions and Corresponding Questions and Answers

We have a certain number of objects: M, a part of which, for lack of a better name, will be called C's. The C's are composed of B's and G's. Each of this latter two types is divided into R's and T's. From the other information available which you will find in the questions, your object will be to discover the number of G's that are also T's.

Questions	Answers
1. Are there C's that are not B's and G's?	1. No.
2. How many B's are C's?	2. Thirty.
3. How many B's are M's?	3. One hundred and twenty.
4. How many C's are R's?	4. Thirty-five.
5. Are there more G's than B's among the M's?	5. Yes.
6. What is the value of k times the C's?	6. Five hundred and fifty.
7. What is the total number of C's?	7. Fifty.
8. How many B's that are C's are also T's?	8. Ten.
9. How many G's that are C's are also R's?	9. Fifteen.
10. What is the value of k?	10. Eleven.

Solution: 5

APPENDIX I

Problem 31 C

Instructions and Corresponding Questions and Answers

Assume that X, A, D, P, and S represent properties among F objects. Not -X, not -A, not -D, and so on represent lack of these properties. The not -X's are composed of not -A's and not -D's. Each of these latter is divided into not -P's and not -S's. From the other information available, which you will find in the questions your object will be to discover the number of not -D's that are also not -S's.

	Questions		Answers
1.	Are there not -X's that are A's and D's?	1.	No.
2.	How many not -A's are F's?	2.	One hundred.
3.	Are there more not -D's than not -A's among the F's?	3.	Yes.
4.	How many not -A's are not -X's?	4.	Fourteen.
5.	What is the total number of not -X's?	5.	Forty.
6.	How many not -X's are not -P's?	6.	Twenty-four.
7.	What is the value of 1 times the not -X's?	7.	Four hundred and forty.
8.	What is the value of 1?	8.	Eleven.
9.	How many not -D's that are not -X's are also not -P's?	9.	Twenty.
10.	How many not -A's that are not -X's are also not -S's?	10.	Ten.

Solution: 6

Problem 31 D

Instructions and Corresponding Questions and Answers

From R objects L has been selected. These L objects are divided into A's and B's. Each of these latter types is divided into M's and N's. From the other information available, which you will find in the questions, your object will be to discover how many N's are also B's.

Questions	Answers
1. How many A's are R's?	1. W.
2. What is the total number of L's?	2. E+F+H+I = P+Q = L
3. How many L's are M's?	3. E+F = X
4. How many A's are L's?	4. E+H = P
5. Are there more B's than A's among the R's?	5. Yes.
6. Are there L's that are not B's and A's?	6. No.
7. How many B's that are L are also M?	7. F.
8. How many A's that are L are also N?	8. H.
9. What is the value of K?	9. T.
10. What is the value of k times the L's?	10. Z.

Solution: I

Problem 35 A

Instructions and Corresponding Questions and Answers

A college choral group is composed of freshmen, sophomores and juniors. The chorus has three voices or parts which are high, medium and low. The questions and answers below give vital information concerning the group. From those facts, you are to find the number of juniors singing the middle or medium part.

Questions	Answers
1. How many juniors are in this college?	1. One thousand, five hundred and sixty-seven.
2. How many freshmen are in the chorus?	2. Twenty-three.
3. How many sophomores are in the middle voice?	3. Ten.
4. How many chorus members are there?	4. Seventy-six.
5. How many girls are in the chorus?	5. Forty-five.
6. How many sophomores are in the chorus?	6. Twenty-eight.
7. How many juniors sing the high voice?	7. Seven.
8. How many freshmen are in this college?	8. One thousand, eight hundred and forty-eight.
9. How many freshmen sign the high voice?	9. Eight.
10. How many low voice members are there?	10. Twenty-eight.
11. How many sophomores sing the high part?	11. Nine.
12. How many pianos does the chorus have?	12. Three.
13. How many freshmen sing the low voice?	13. Nine.
14. How many chorus members sing the high voice?	14. Twenty-four.
15. How many juniors are in the low voice section?	15. Ten.
16. How many freshmen sing the middle part?	16. Six.

Problem 35 A
(Continued)

Questions	Answers
17. How many sophomores sing the low part?	17. Nine.

Solution: 8

APPENDIX I

Problem 35 B

Instructions and Corresponding Questions and Answers

T objects are composed only of M, N and P types. Each of these latter three types is divided into Q's, R's, and S's. From the questions and answers, you can discover the various relationships of these objects. Make use of this available information to determine how many of the T objects are N's and also S's.

Questions	Answers
1. How many S's are A's?	1. Three hundred and fifty.
2. How many Q's are there among the T's?	2. Nineteen.
3. How many C's are there among the T's?	3. Forty-three.
4. How many R's are also N's?	4. Eight.
5. What is the total number of T objects?	5. Sixty-three.
6. How many P's are there among the T's?	6. Twenty-one.
7. How many R's are there among the T's?	7. Twenty-four.
8. How many Q's are also M's?	8. Five.
9. How many R's are also M's?	9. Ten.
10. How many S's are also M's?	10. Two.
11. How many Q's are A's?	11. Four hundred.
12. How many R's are also P's?	12. Six.
13. How many Q's are also N's?	13. Three.
14. How many S's are also P's?	14. Four.
15. How many M's are among the T's?	15. Seventeen.
16. How many Q's are also P's?	16. Eleven.
17. How many H's are among the A's?	17. Two.

Solution: 14

Problem 35 C

Instructions and Corresponding Questions and Answers

A class of objects is distinguished by calling some B's and some not -B's, depending on the possession or non-possession of a certain property. The not -B's are further distinguished into not -X's, not -Y's and not -Z's. Each of these latter is divided into not -D's, not -E's, and not -F's. From the accompanying questions and answers, you can discover the relationships that exist between these objects. Make use of the information available to determine how many not -B objects are not -Y's and also not -F's.

	Questions		Answers
1.	How many not -D's are not -A's?	1.	One hundred and fifty.
2.	How many not -F's are also not -X's?	2.	Seven.
3.	How many not -E's are there among the not -B's?	3.	Fifteen.
4.	How many not -G's are there among the not -B's?	4.	Thirty.
5.	What is the total number of not -B's?	5.	Forty-five.
6.	How many not -E's are also not -Y's?	6.	Six.
7.	How many not -D's are there among the not -B's?	7.	Six.
8.	How many not -F's are not -A's?	8.	One hundred.
9.	How many not -E's are also not -Z's?	9.	Five.
10.	How many not -D's are also not -Y's?	10.	Two.
11.	How many not -F's are also not -Z's?	11.	Nine.
12.	How many not-X's are there among the not -B's?	12.	Twelve.
13.	How many not -D's are also not -Z's?	13.	Three.
14.	How many not -H's are there among the not -A's?	14.	$2 \log \cos 30^\circ$.
15.	How many not -E's are also not -X's?	15.	Four.

Problem 35 C
(Continued)

Questions	Answers
16. How many not -Z's are there among the not -B's?	16. Seventeen.
17. How many not -D's are also not -X's?	17. One.

Solution: 8

APPENDIX I

Problem 35 D

Instructions and Corresponding Questions and Answers

A group of L objects is composed of A, B, and C types. Each of these latter types is divided into D's, E's, and F's. From the questions and answers, you can discover the various relationships of these objects. Make use of this available information to determine how many L objects are B's and also F's.

Questions	Answers
1. How many F's are in J?	1. U.
2. How many L's are D's?	2. M + N + O = X.
3. What is the number of L's?	3. M+N+O+R+Q+P+S+T+V = X+Y+Z = G+H+I = L.
4. How many E's are B's?	4. Q.
5. How many L's are K's?	5. W.
6. How many D's are in M?	6. X-M+O.
7. How many L's are E's?	7. R + Q + P = Y.
8. How many F's are A's?	8. S.
9. How many E's are A's?	9. R.
10. How many D's are A's?	10. M.
11. How many L's are C's?	11. O + P + V = I.
12 How many F's are C's?	12. V.
13. How many L's are A's?	13. M + R + S = G.
14. How many D's are C's?	14. O.
15. How many U's are M's?	15. U - J.
16. How many D's are B's?	16. N.
17. How many E's are C's?	17. P.

Solution: T

Problem 36 C

Instructions and Corresponding Questions and Answers

Each of the areas in the accompanying figure are identified by means of the numbers found printed in them. These numbers are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

The problem then is this. Each area has been assigned a symbol other than the above identifying numbers according to a predetermined plan. Your task will be to discover the symbols belonging to the areas which have their identifying numbers circled. This may be accomplished by asking the symbols for any area other than the ones circled. Decide which area you would like to know first. Then ask for its symbol. Continue working in this fashion until you feel you have sufficient information to specify accurately the symbols for the areas with the circled numbers. Remember, you may ask as many questions you feel necessary to find a solution, but do not ask more questions than you need.

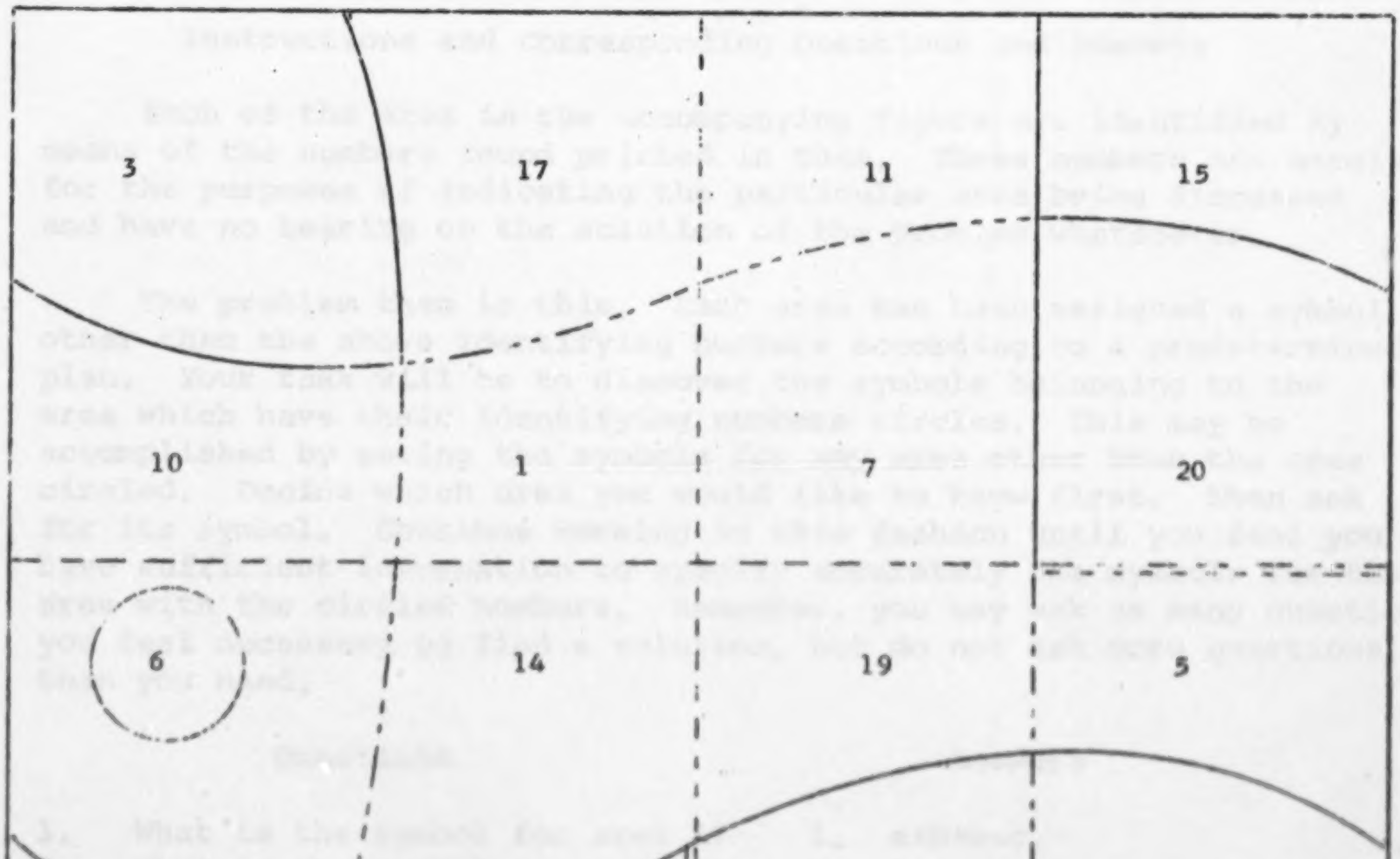
	Questions		Answers
1.	What is the symbol for area 1?	1.	2b+2f.
2.	What is the symbol for area 3?	2.	2a+2c.
3.	What is the symbol for area 4?	3.	2a+c+f.
4.	What is the symbol for area 5?	4.	a+c+d+e.
5.	What is the symbol for area 7?	5.	a+2b+f.
6.	What is the symbol for area 8?	6.	a+c+e+f.
7.	What is the symbol for area 9?	7.	3a+f.
8.	What is the symbol for area 10?	8.	a+b+c+f.
9.	What is the symbol for area 11?	9.	2a+b+f.
10.	What is the symbol for area 12?	10.	2a+c+d.
11.	What is the symbol for area 13?	11.	a+b+2d.
12.	What is the symbol for area 14?	12.	2b+c+f.
13.	What is the symbol for area 15?	13.	3a+c.
14.	What is the symbol for area 16?	14.	2a+b+f.
15	What is the symbol for area 17?	15.	a+b+c+f.

Problem 36 C
(Continued)

	Questions		Answers
16.	What is the symbol for area 18?	16.	3a+d.
17.	What is the symbol for area 19?	17.	2b+c+d.
18.	What is the symbol for area 20?	18.	2a+c+e.

Solution: A+B+C+F and C+2D+E

PROBLEM 36 C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Red Solid Line

Red Broken Line

Apoendix I

Problem 36 D

Instructions and Corresponding Questions and Answers

Each of the area in the accompanying figure are identified by means of the numbers found printed in them. These numbers are merely for the purposes of indicating the particular area being discussed and have no bearing on the solution of the problem whatsoever.

The problem then is this. Each area has been assigned a symbol other than the above identifying numbers according to a predetermined plan. Your task will be to discover the symbols belonging to the area which have their identifying numbers circles. This may be accomplished by asking the symbols for any area other than the ones circled. Decide which area you would like to know first. Then ask for its symbol. Continue working in this fashion until you feel you have sufficient information to specify accurately the symbols for the area with the circled numbers. Remember, you may ask as many questio you feel necessary to find a solution, but do not ask more questions than you need.

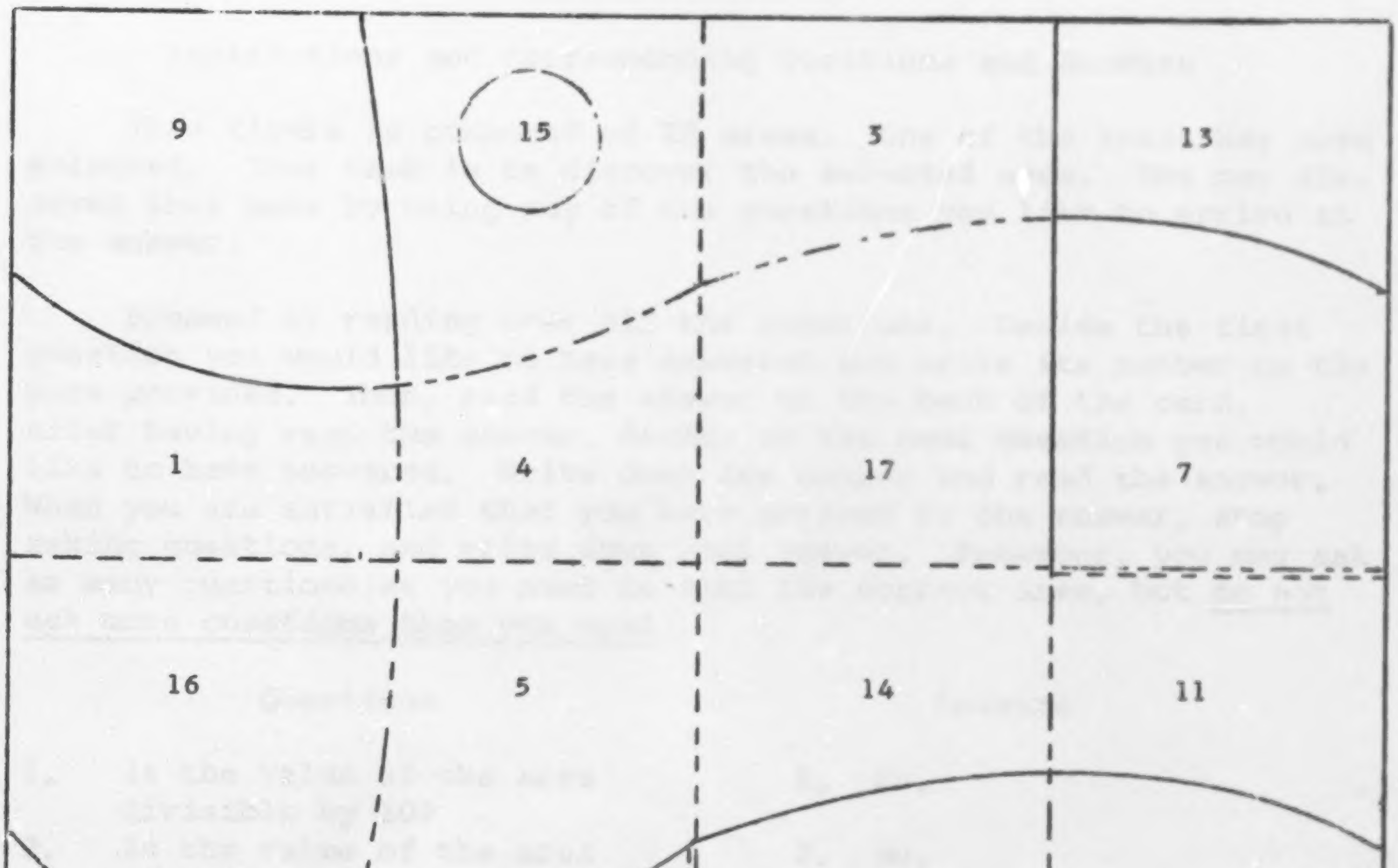
Questions	Answers
1. What is the symbol for area 1?	1. a+b+c-c.
2. What is the symbol for area 2?	2. 2a+c-c.
3. What is the symbol for area 3?	3. 2a+b-c.
4. What is the symbol for area 4?	4. 2b-2c.
5. What is the symbol for area 5?	5. 2b+c-c.
6. What is the symbol for area 6?	6. 3a-a.
7. What is the symbol for area 7?	7. 2a+c-b.
8. What is the symbol for area 9?	8. 2a+2c.
9. What is the symbol for area 10?	9. 2a+b-c.
10. What is the symbol for area 11?	10. a+c-a-b.
11. What is the symbol for area 12?	11. 3a-c.
12. What is the symbol for area 13?	12. 3a+c.
13. What is the symbol for area 14?	13. 2b+c-a.
14. What is the symbol for area 16?	14. a+b+c-c.
15. What is the symbol for area 17?	15. a+2b-c.

Problem 36 D
(Continued)

Questions		Answers	
16.	What is the symbol for area 18?	16.	a+b-2a.
17.	What is the symbol for area 19?	17.	a+c-b-c.
18.	What is the symbol for area 20?	18.	2a+c-a.

Solution: A+B+C-C and C-2A-B

PROBLEM 36 D


= Red Solid Line
= Red Broken Line

Problem 40

Instructions and Corresponding Questions and Answers

This figure is composed of 20 areas. One of the areas has been selected. Your task is to discover the selected area. You may discover this area by using any of the questions you like to arrive at the answer.

Proceed by reading over all the questions. Decide the first question you would like to have answered and write its number on the page provided. Then, read the answer on the back of the card. After having read the answer, decide on the next question you would like to have answered. Write down its number and read the answer. When you are satisfied that you have arrived at the answer, stop asking questions, and write down your answer. Remember, you may ask as many questions as you need to find the correct area, but do not ask more questions than you need.

Questions	Answers
1. Is the value of the area divisible by 10?	1. No.
2. Is the value of the area divisible by 2?	2. No.
3. Is the value of the area divisible by both 2 and 3?	3. No.
4. Is the value of the area divisible by 4?	4. No.
5. Is the value of the area divisible by 9?	5. No.
6. Is the value of the area divisible by both 2 and 4?	6. No.
7. Is the value of the area divisible by 3?	7. No.
8. Is the value of the area divisible by 6?	8. No.
9. Is the value of the area divisible by 5?	9. No.
10. Is the value of the area divisible by 7?	10. No.

Solution: 11

PROBLEM 40

20	9	4	25
25	2	5	6
8	21	10	39
35	11	27	12
16	14	18	5

Problem 42

Instructions and Corresponding Questions and Answers

This figure is composed of 24 areas. The numbers in the areas are merely for the purpose of identifying a particular area and have no bearing on the solutions of the problem whatsoever.

One of the areas has been selected. Your task is to discover the selected area. You may discover this area by using any of the questions you like to arrive at the answer.

Proceed by reading over all the questions. Decide the first question you would like to have answered and write its number on the page provided. Then, read the answer on the back of the card. After having read the answer, decide on the next question you would like to have answered. Write down its number and read the answer. When you are satisfied that you have arrived at the answer, stop asking questions, and write down your answer. Remember, you may ask as many questions as you need to find the correct area, but do not ask more questions than you need.

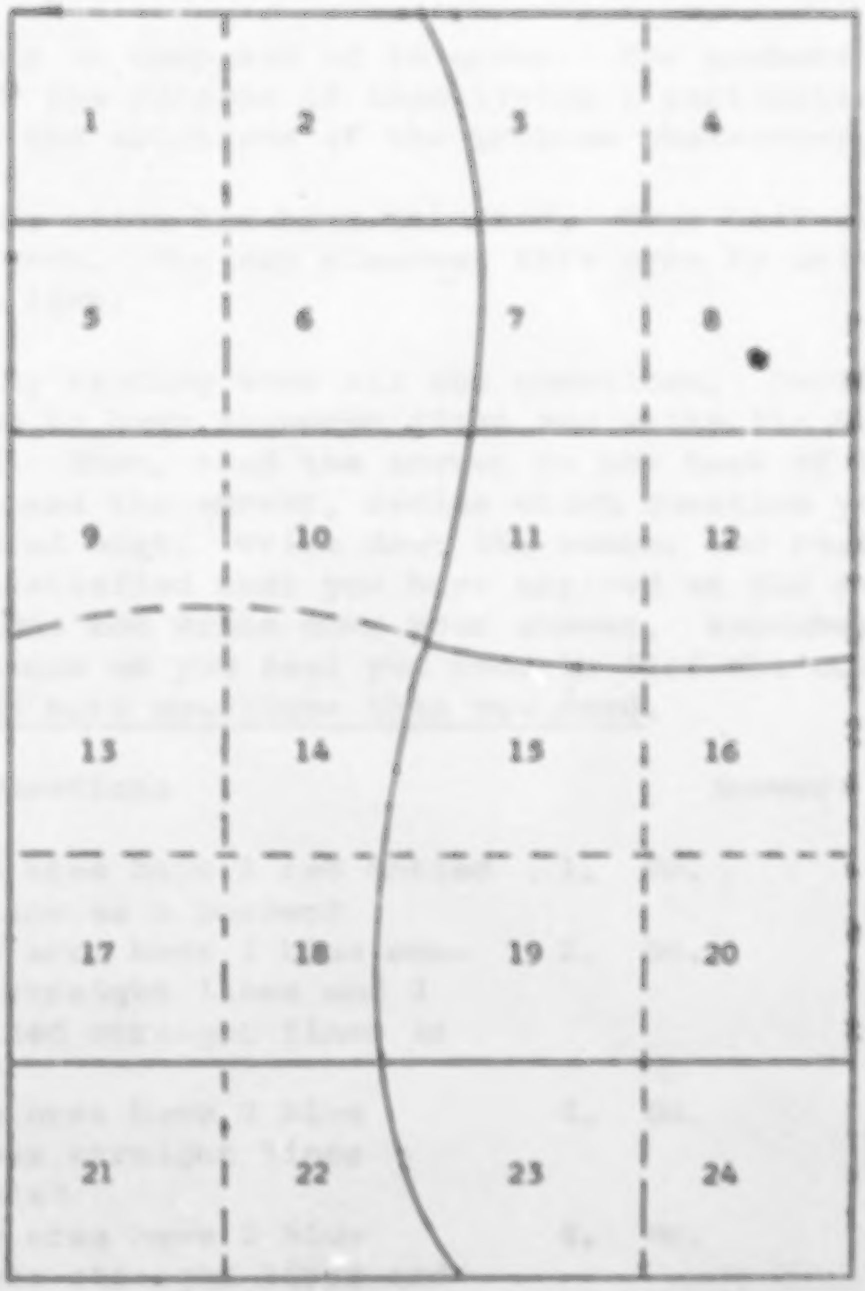
Questions	Answers
1. Is it above the unbroken curve line?	1. No.
2. Does it have 2 curved lines as borders?	2. No.
3. Is it to the right of the vertical curve line?	3. Yes.
4. Does it have 2 continuous straight lines and 2 broken lines as borders?	4. No.
5. Does it have 2 broken straight line borders?	5. No.
6. Does it have any combinations of 2 broken and 2 curved sides?	6. No.
7. Is it below the dotted curve line?	7. No.
8. Does it have 3 continuous straight lines and 1 broken straight line as borders?	8. No.

Problem 42
(Continued)

Questions	Answers
9. Does it have a broken curved line as a border?	9. No.
10. Does it have at least 1 continuous straight line and 2 continuous curved lines as borders?	10. No.

Solution: 23

PROBLEM 42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Problem 44 A

Instructions and Corresponding Questions and Answers

The figure is composed of 16 areas. The numbers in the areas are merely for the purpose of identifying a particular area and have no bearing on the solutions of the problem whatsoever.

One of the areas has been selected. Your task is to discover the selected area. You may discover this area by using any of the questions you like.

Proceed by reading over all the questions. Decide which questio you would like to have answered first and write its number on the page provided. Then, read the answer on the back of the card. After having read the answer, decide which question you would like to have answered next. Write down its number and read the answer. When you are satisfied that you have arrived at the answer, stop asking questions and write down your answer. Remember, you may ask as many questions as you feel you need to find the correct area, but do not ask more questions than you need.

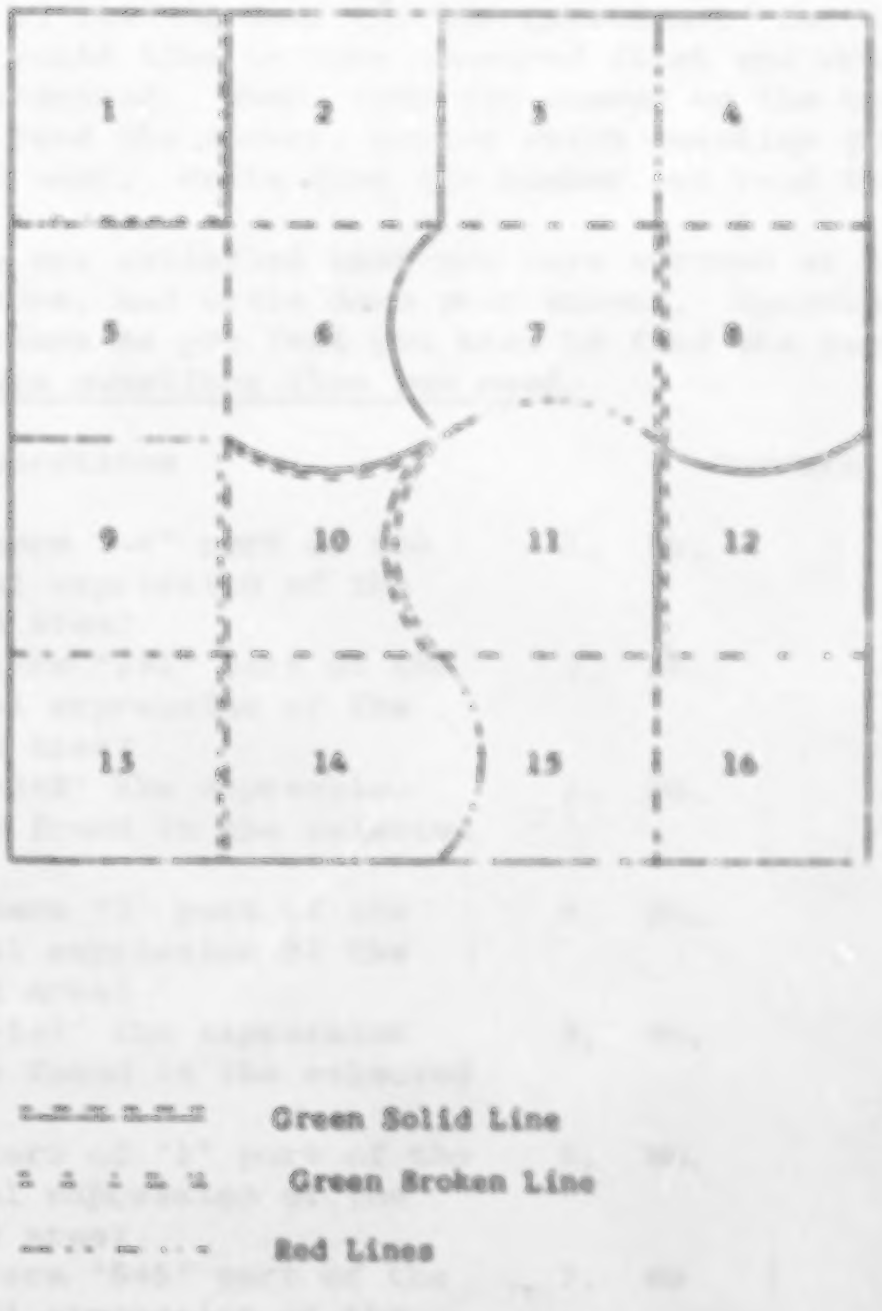
Questions	Answers
1. Does the area have 1 red dotted curved line as a border?	1. No.
2. Does the area have 2 blue continuous straight lines and 2 blue dotted straight lines as borders?	2. No.
3. Does the area have 2 blue continuous straight lines as borders?	4. No.
4. Does the area have 2 blue continuous straight lines and 2 green continuous straight lines as borders?	4. No.
5. Does the area have 1 greeen continuous curved line as a border?	5. No.
6. Does the area have 2 green continuous straight lines as borders?	6. No.

Problem 44 A
(Continued)

Questions	Answers
7. Does the area have 1 blue continuous straight line as a border?	7. No.
8. Does the area have 4 blue lines as borders?	8. No.
9. Does the area have 1 green continuous straight line as a border?	9. No.
10. Does the area have 1 red dotted straight line as a border?	10. No.

Solution: 7

PROBLEM 44 A


Green Solid Line
Green Broken Line
Red Lines

Problem 44 B

Instructions and Corresponding Questions and Answers

The figure is composed of 16 areas. Each area is represented by a numerical expression. One of the areas has been selected. Your task is to discover the selected area. You may discover this area by using any of the questions you like.

Proceed by reading over all the questions. Decide which question you would like to have answered first and write its number on the page provided. Then, read the answer on the back of the card. After having read the answer, decide which question you would like to have answered next. Write down its number and read the answer.

When you are satisfied that you have arrived at the answer, stop asking questions, and write down your answer. Remember, you may ask as many questions as you feel you need to find the correct area, but do not ask more questions than you need.

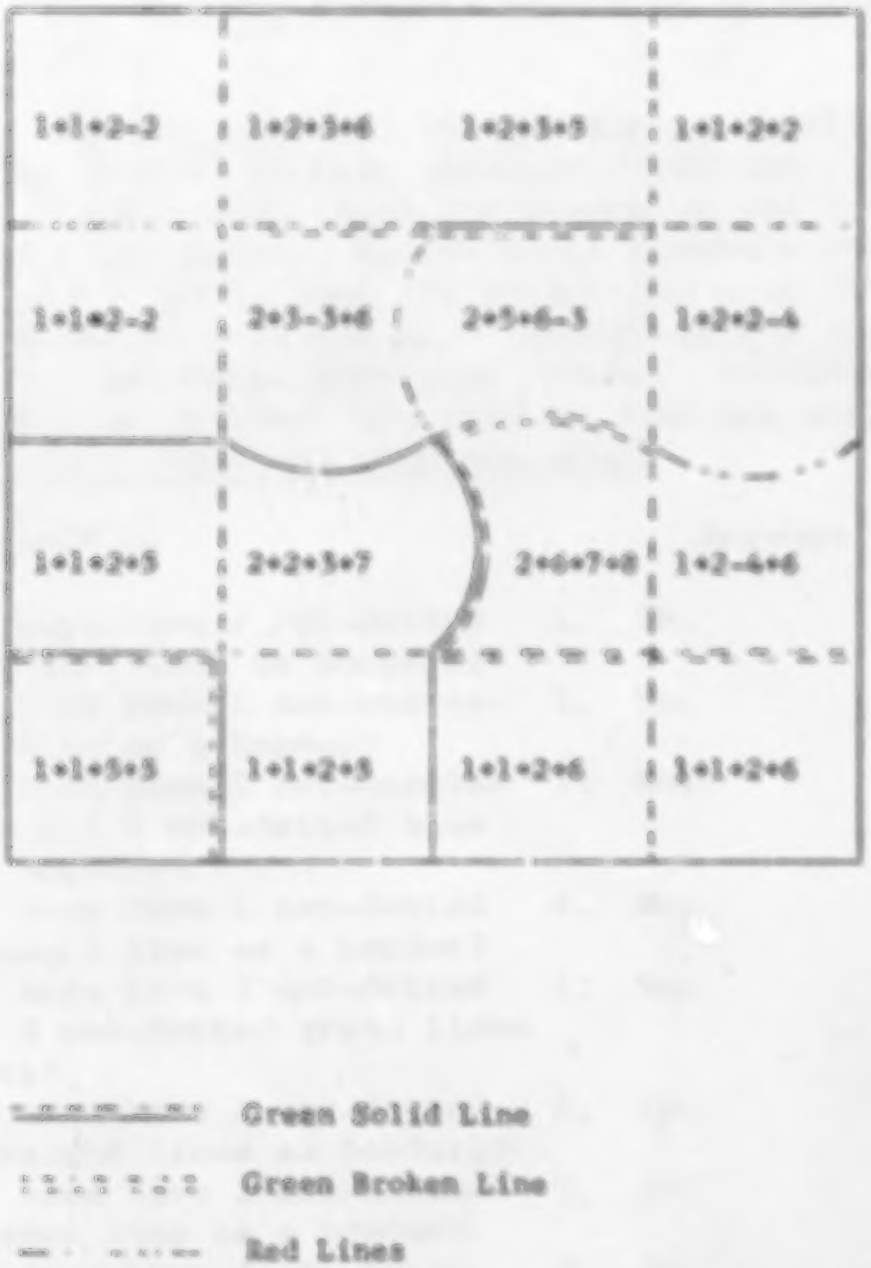
Questions	Answers
1. Is the term "-4" part of the numerical expression of the selected area?	1. No.
2. Is the term "1+1" part of the numerical expression of the selected area?	2. No.
3. Is "1+1+2+2" the expression which is found in the selected area?	3. No.
4. Is the term "7" part of the numerical expression of the selected area?	4. No.
5. Is "1+1+5+5" the expression which is found in the selected area?	5. No.
6. Is the term of "1" part of the numerical expression of the selected area?	6. No.
7. Is the term "5+5" part of the numerical expression of the selected area?	7. No

Problem 44 B
(Continued)

Questions	Answers
8. Is the term "5" part of the numerical expression of the selected area?	8. No.
9. Are all the terms of the expression positive numbers from 1 to 4?	9. No.
10. Is "1+1+2-2" the expression which is found in the selected area?	10. No.

Solution: 2+3-3+6

PROBLEM 44 B


1+1+2-2
1+2+3+6
1+2+3+5
1+1+2+2
1+1+2-2
2+3-3+6
2+5+6-3
1+2+2-4
1+1+2+5
2+2+3+7
2+6+7+8
1+2-4+6
1+1+5+5
1+1+2+5
1+1+2+6
1+1+2+6
Green Solid Line
Green Broken Line
Red Lines

Appendix I

Problem 44 C

Instructions and Corresponding Questions and Answers

The figure is composed of 16 areas. The numbers in the areas are merely for the purpose of identifying a particular area and have no bearing on the solutions of the problem whatsoever.

One of the areas has been selected. Your task is to discover the selected area. You may discover this area by using any of the questions you like.

Proceed by reading over all the questions. Decide which questions you would like to have answered first and write its number on the page provided. Then, read the answer on the back of the card. After having read the answer, decide which question you would like to have answered next. Write down its number and read the answer. When you are satisfied that you have arrived at the answer, stop asking questions, and write down your answer. Remember, you may ask as many questions as you feel you need to find the correct area, but do not ask more questions than you need.

Questions	Answers
1. Does the area have 2 not-dotted blue straight lines as borders?	1. No.
2. Does the area have 1 not-continuous red line as a border?	2. No.
3. Does the area have 2 not-continuous blue and 2 not-dotted blue lines as borders?	3. No.
4. Does the area have 1 not-dotted blue straight line as a border?	4. No.
5. Does the area have 2 not-dotted blue and 2 not-dotted green lines as borders?	5. No.
6. Does the area have 2 not-dotted green straight lines as borders?	6. No.
7. Does the area have 1 not-dotted green curved line as a border?	7. No.
8. Does the area have 4 not-green lines as borders?	8. No.

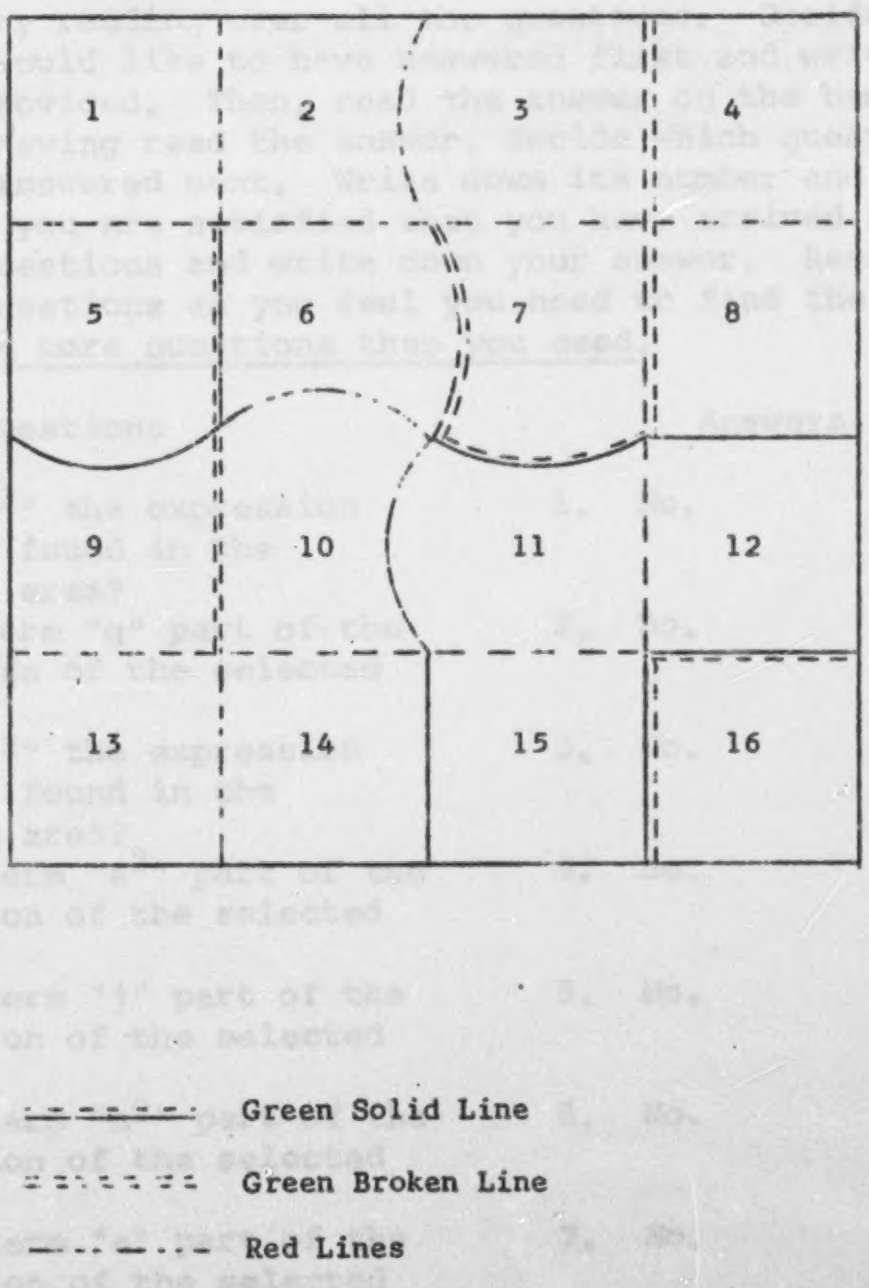
Problem 44 C
(Continued)

	Questions		Answers
9.	Does the area have 1 not-continuous red straight line as a border?	9.	No.
10.	Does the area have 1 not-dotted green straight line as a border?	10.	No.

Solution: 10

APPENDIX I

PROBLEM 44 C


Green Solid Line
Green Broken Line
Red Lines

Problem 44 D

Instructions and Corresponding Questions and Answers

The figure is composed of 16 areas. Each area is represented by an algebraic expression. One of these areas has been selected. Your task is to discover the selected area. You may discover the area by using any of the questions you like.

Proceed by reading over all the questions. Decide which question you would like to have answered first and write its number on the page provided. Then, read the answer on the back of the card. After having read the answer, decide which question you would like to have answered next. Write down its number and read the answer. When you are satisfied that you have arrived at the answer, stop asking questions and write down your answer. Remember, you may ask as many questions as you feel you need to find the correct area, but do not ask more questions than you need.

Questions	Answers
1. Is "$a^2.b^2$" the expression which is found in the selected area?	1. No.
2. Is the term "g" part of the expression of the selected area?	2. No.
3. Is "$a^2.h^2$" the expression which is found in the selected area?	3. No.
4. Is the term "a^2" part of the expression of the selected area?	4. No.
5. Is the term "j" part of the expression of the selected area?	5. No.
6. Is the term "h^2" part of the expression of the selected area?	6. No.
7. Is the term "a" part of the expression of the selected area?	7. No.

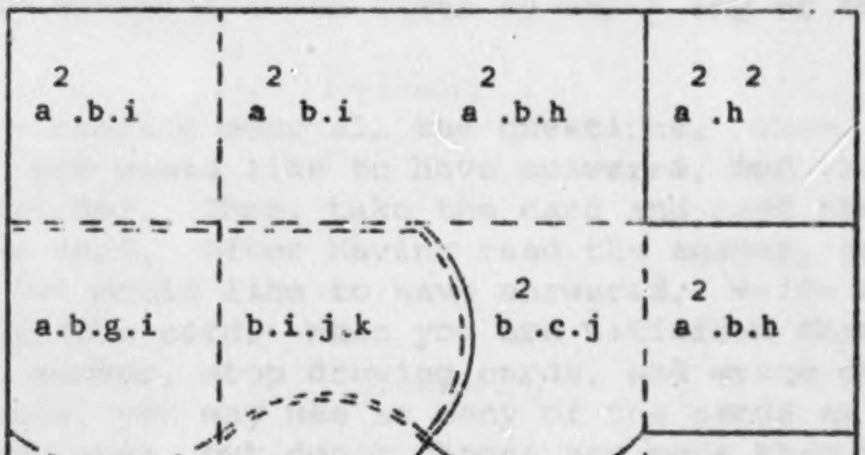
Problem 44 D
(Continued)

	Questions		Answers
8.	Is "$a^2.b.e$" the expression which is found in the selected area?	8.	No.
9.	Is the term "h" part of the expression of the selected area?	9.	No.
10.	Is the expression of the selected area composed of a's and b's?	10.	No.

Solution: bcfi

36.

PROBLEM 44 D


$a^2.b.i$
$a^2 b.i$
$a^2.b.h$
$a^2.h^2$
a.b.g.i
b.i.j.k
$b^2.c.j$
$a^2.b.h$
$a^2.b. .g$
b.f.h.k
b.c.f.i.
$a^2.b.e$
$a^2.b^2$
a.b.c.h.
a.b.c.i
$a^2.b.e$
Green Solid Line
Green Broken Line
Red Lines

Appendix I

Problem 46

Instructions and Corresponding Questions and Answers

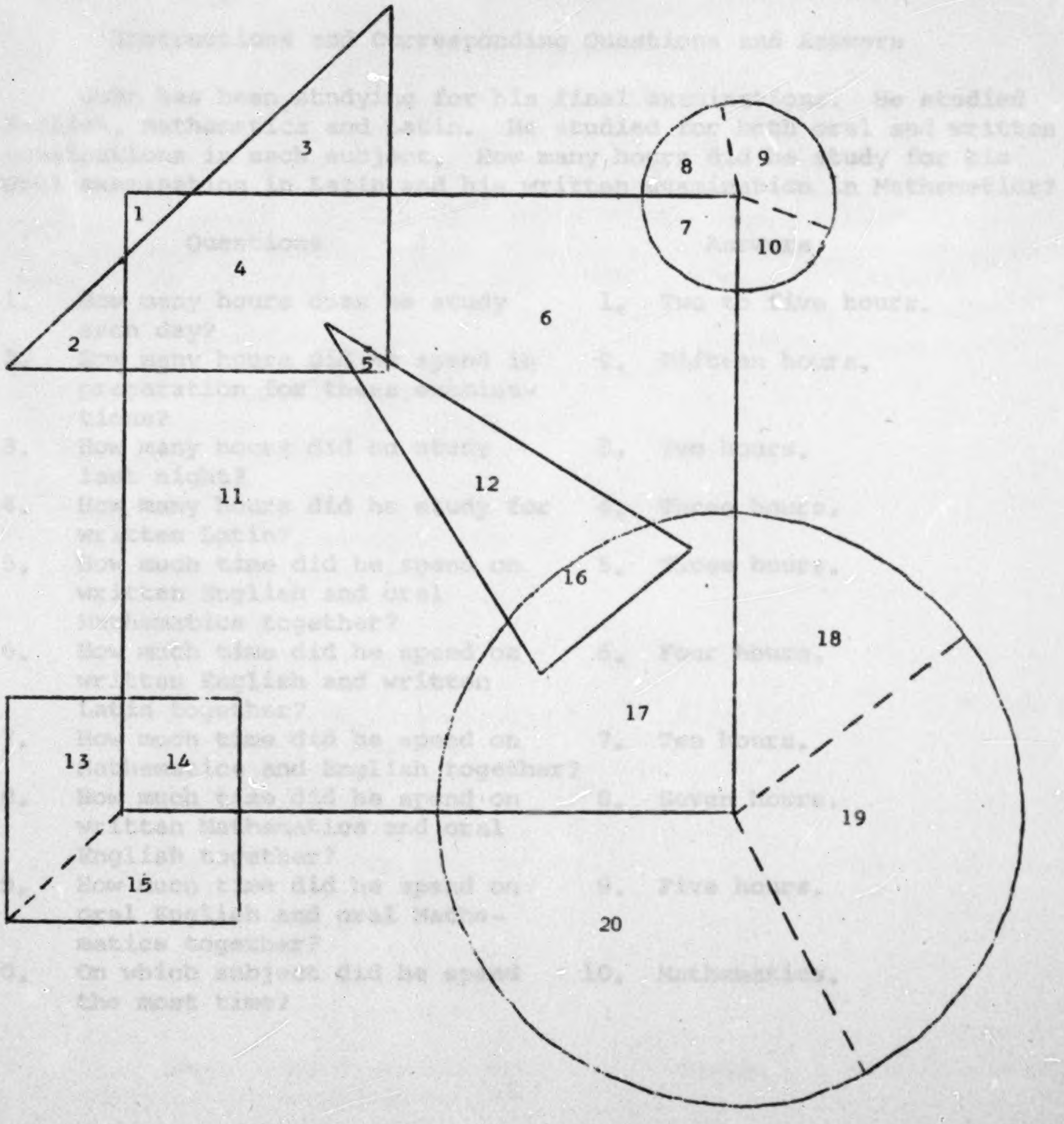
The figure here is composed of overlapping geometric figures and lines, forming twenty areas. You will notice that an area is any enclosed part of the figure that does not have a line through it. One of the areas has been selected. Your task is to find out which one it was. To find the particular area, you must discover enough facts about it so that it can be distinguished from other areas. You may discover these facts by using any of the questions you like.

Proceed by reading over all the questions. Then, decide on the first question you would like to have answered, and write its number of the page provided. Then, take the card and read the answer on the back of the card. After having read the answer, decide on the next question you would like to have answered. Write down its number, and take the card. When you are satisfied that you have arrived at the answer, stop drawing cards, and write down your answer. Remember, you may use as many of the cards as you need to find the correct area, but don't choose any more than you need.

	Questions		Answers
1.	Is it a three-sided area with 3 straight sides?	1.	No.
2.	Is it inside the large square?	2.	Yes.
3.	Does it have 2 broken lines as borders?	3.	No.
4.	Does it have a broken line as a border?	4.	No.
5.	Is it in the small circle?	5.	No.
6.	Is it in the small square?	6.	No.
7.	Is it in only 2 figures?	7.	No.
8.	Is it in only 1 figure?	8.	No.
9.	Does it have only 4 straight lines as borders?	9.	No.
10.	Is it a three-sided area with 2 straight sides and 1 curve side?	10.	No.

Solution: 16

PROBLEM 46



Instructions and Corresponding Questions and Answers

John has been studying for his final examinations. He studied English, Mathematics and Latin. He studied for both oral and written examinations in each subject. How many hours did he study for his oral examination in Latin and his written examination in Mathematics?

Questions	Answers
1. How many hours does he study each day?	1. Two to five hours.
2. How many hours did he spend in preparation for these examinations?	2. Fifteen hours.
3. How many hours did he study last night?	3. Two hours.
4. How many hours did he study for written Latin?	4. Three hours.
5. How much time did he spend on written English and oral Mathematics together?	5. Three hours.
6. How much time did he spend on written English and written Latin together?	6. Four hours.
7. How much time did he spend on Mathematics and English together?	7. Ten hours.
8. How much time did he spend on written Mathematics and oral English together?	8. Seven hours.
9. How much time did he spend on oral English and oral Mathematics together?	9. Five hours.
10. On which subject did he spend the most time?	10. Mathematics.

Solution: 2+4=6

Appendix I

Problem 59 A

Instructions and Corresponding Questions and Answers

John has been studying for his final examinations. He studied English, Mathematics and Latin. He studied for both oral and written examinations in each subject. How many hours did he study for his oral examination in Latin and his written examination in Mathematics?

Questions	Answers
1. How many hours does he study each day?	1. Two to five hours.
2. How many hours did he spend in preparation for these examinations?	2. Fifteen hours.
3. How many hours did he study last night?	3. Two hours.
4. How many hours did he study for written Latin?	4. Three hours.
5. How much time did he spend on written English and oral Mathematics together?	5. Three hours.
6. How much time did he spend on written English and written Latin together?	6. Four hours.
7. How much time did he spend on Mathematics and English together?	7. Ten hours.
8. How much time did he spend on written Mathematics and oral English together?	8. Seven hours.
9. How much time did he spend on oral English and oral Mathematics together?	9. Five hours.
10. On which subject did he spend the most time?	10. Mathematics.

Solution: 2+4=6

APPENDIX I

Problem 59 B

Instructions and Corresponding Questions and Answers

A number of objects have been collected into a general classification S. The S objects are composed of E, M, and L types. Each one of these types are further subdivided into W's and O's. From the questions and answers you can discover the various relationships of these objects. Make use of this available information to determine how many S objects that are L's are also O's and also determine which of the S objects that are M's and also W's.

Questions	Answers
1. How many other general designations are there beside S?	1. Four.
2. How many objects with another designation are N's?	2. Fifteen.
3. How many S's are there?	3. Thirty.
4. What is the number of S objects that are E and also W's together with the S's that are M's and also O's?	4. Seven.
5. How many S objects are L's and are also W's?	5. Five.
6. What is the number of S objects that are M's and also W's together with the S objects that are E's and also O's?	6. Thirteen.
7. What is the number of S objects that are E's and also O's together with the S objects that are L's and also O's?	7. Eleven.
8. What is the number of S objects that are E and also W's together with the S's that are L and also W's?	8. Nine.
9. What is the number of S objects that are M's together with the S objects that are E's?	9. Twenty.
10. What type has the largest number of objects?	10. All types have the same number of objects.

Solution: 5+7=12

Problem 59 C

Instructions and Corresponding Questions and Answers

A class of objects is distinguished by calling some X's and some others not -X's depending on the possession or non-possession of certain properties. The not -X's are further distinguished into not -A's, not -B's and not -C's. Each of these latter is also a not -D or a not -E. From the accompanying questions and answers you can discover the relationships that exist between these objects. Make use of the information available to determine how many not -X objects that are not -B's are also not -D's and also determine which of the not -X's that are not -C's are also not -E's.

Questions	Answers
1. How many other general designations are there besides the not -X's?	1. Three.
2. What is the number of not -X objects that are not -A and also not -D together with the not -X's that are not -B and also not -E?	2. Thirteen.
3. What is the number of not -X objects that are not -A's together with the not -X's that are not -B's?	3. Twenty-two.
4. What is the number of not -X objects that are not -B's and also not -D's together with the not -X's that are not -A's and also not -E's?	4. Nine.
5. How many not -X's are there?	5. Thirty.
6. What is the number of not -X objects that are not -A's and also not -E's together with the not -X's that are not -B's and also not -E's?	6. Eleven.
7. How many objects with another designation are not -P's?	7. Fifty.
8. How many not -X objects are not -C's that are also not- D's?	8. Five.

Problem 59 C
(Continued)

Questions	Answers
9. What is the number of not -X objects that are not -A's and also not -D's together with the not -X's that are not -C's and also not -D's?	9. Eight.
10. What type has the largest number of objects?	10. Not -B's.

Solution: 3+8=11

APPENDIX I

Problem 59 D

Instructions and Corresponding Questions and Answers

A number of objects have been collected into a general classification A. These A objects are composed of N, O, and P types. Each one of this type is further subdivided into S's and Q's. From the questions and answers, you can discover the various relationships of these objects. Make use of this available information to determine how many A's that are P's are also S's and also determine which of the A's that are O's and also Q's.

	Questions		Answers
1.	How many F's are M's?	1.	K+J.
2.	How many A's are N's and also S's together with the A's that are O's and also Q's?	2.	a+d.
3.	How many A's are B's?	3.	L.
4.	How many A's are there?	4.	X+Y+Z = a+b+c+d+e+f.
5.	How many A's are P's and also Q's?	5.	f.
6.	How many A's are O's that are S's together with the A's that are N's and also S's?	6.	a+c=C
7.	How many A's are N's and also Q's together with the A's that are P's and also Q's?	7.	b+f.
8.	How many A's are O's and N's?	8.	X+Y = a+b+c+d.
9.	What type has the largest number of objects?	9.	All the types have the same number of objects.
10.	How many A's are N's that are Q's together with the A's that are O's and also S's?	10.	b+c.

Solution: E+D

APPENDIX I

TABLE 2

Number of Correct Solutions in the High School

Problem	Group I	Group II	Group III	Group IV
31A	5	3	5	1
31B	4	2	2	0
31C	5	2	0	0
31D	5	2	1	0
40	3	5	5	4
42	4	3	3	2
36C	3	1	1	0
36D	4	0	1	0
35A	5	5	4	2
35B	5	1	2	2
35C	5	2	3	0
35D	4	3	3	0
44A	5	4	5	1
44B	5	5	5	3
44C	5	3	2	1
44D	5	5	5	5
59A	4	1	2	0
59B	4	0	0	0
59C	3	0	1	0
59D	3	0	0	0
46	4	1	1	0
25	5	3	5	2
Σ	95	51	55	23

Chi Squares for the Total Number of Correct Answers Between Groups in High School

I-II=39.42***
I-III=33.52***
I-IV=94.75***
II-III=.29
II-IV=13.25***
III-IV=20.33***

(I+II)-(III+IV)=42.05***

(I+III)-(II+IV)=52.50***

***p<.001

Appendix II

TABLE 2

Number of Correct Solutions in the College

Problem	Group I	Group II	Group III	Group IV
31A	5	4	5	3
31B	5	5	3	2
31C	4	4	4	2
31D	3	4	2	0
40	5	4	4	5
42	5	5	5	4
36C	5	2	1	1
36D	4	3	3	0
35A	4	5	5	4
35B	4	4	4	0
35C	5	4	3	2
35D	4	4	3	1
44A	5	5	4	5
44B	5	4	5	4
44C	5	5	4	4
44D	5	5	4	5
59A	5	4	3	3
59B	4	5	3	1
59C	5	4	3	0
59D	5	4	3	0
46	5	4	4	2
25	4	4	4	5
Σ	101	92	79	53

Chi Squares for the Total Number of Correct Answers Between Groups in College

I-II=3.41*
I-III=14.81***
I-IV=49.87***
II-III=4.43*
II-IV=30.76***
III-IV=12.80***

(I+II)-(III+IV)=43.80***

(I+III)-(II+IV)=14.85***

*=$p<.05$
***$p<.001$

Appendix II

APPENDIX III

AN APPRAISAL OF THREE SCORING PROCEDURES AS DISCRIMINATORS BETWEEN GOOD AND POOR PROBLEM SOLVERS

BY

James B. Erdmann

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CHAPTER I

INTRODUCTION

The last twenty years, and especially the last decade, have witnessed a tremendous interest in a particular area of cognition, namely the problem solving situation. These attempts to qualitatively describe and quantitatively evaluate the problem solving situation have led researchers to employ various instruments and techniques.

The Loyola Psychometric Laboratory has, since its inception, been deeply concerned with this area of research. Here several instruments have been developed, as well as various methods of quantitatively evaluating performance on these instruments. The research presented in this paper concerns itself with a specific type of instrument and with three different scoring techniques developed for the analysis of problem solving performance on this instrument.

CHAPTER II

DEFINITION OF THE PROBLEM

Hand in hand with the development of any instrument must coincide the development of a technique to evaluate performance on that instrument. In the present situation such a technique was not lacking. As a matter of fact, several different techniques or scoring procedures were forthcoming in an attempt to analyze performance on this type of problem or instrument. The next point of concern, given the problem and several methods of evaluation, is which method best evaluates or discriminates performance in the problem. It is on this last point that the present research focuses.

Of the various techniques available for evaluating performance on this instrument, three which have recently been gaining prominence were selected for investigation and critical appraisal.

The specific aim of this research, then, is to analyze the three scoring procedures and determine which is the best discriminator between good and poor problem solvers.

At this point, it is important to note that this evaluation of good or poor problem solving ability is based on the process of the individual problem solver and not the accuracy of his solution. Each of these methods, then, attempts to characterize quantitatively the process of a problem solver using this type of instrument. The question, then, is which quantitative method of characterizing the process best discriminates between good and poor problem solvers.

The subjects will be divided into two groups, an upper and lower, on the basis of criteria purporting to measure problem solving ability. Assuming the instrument measures problem solving ability, it would allow a judgment as to which scoring procedure would best distinguish the two groups and, thus, be the best discriminator between good and poor problem solvers.

CHAPTER III

REVIEW OF RELATED LITERATURE

A. General Considerations:

About 1945 the work of Max Wertheimer in the field of problem solving began and this ignited a vigorous interest in the topic. For him a problem situation has a goal, obstacles to the goal, and no clear perception of the means to reach the goal. An individual's approach to the problem determines whether he is a productive or reproductive thinker. Wertheimer (30) describes reproductive thinking as a mere reproduction of past experience. It is mostly passive in nature. Productive thinking, on the other hand, is active. It demands a mental struggle, a recentering, reorganizing, and restructuring. This last concept, it will be seen later, the pulling-out method attempts to account for. The results of productive thinking, then, is a new product, not just a reproduction of past learning. Wertheimer's approach was a qualitative one. He did not attempt quantification in characterizing the process.

Duncan (5) has a related idea of the problem solving process. He too finds various possible approaches to a problem. For him, however, real problem solving differs from rote learning or conditioning. He would like to visualize a dimension with discovery at one end and conditioning on the other. Problem solving should be high on the discovery end of the dimension.

Bloom and Broder (1) presented another qualitative study. Their methods were similar to Wertheimer's in that they used retrospection and introspection, i.e., reports of the subject. They did introduce a new technique and that was the use of tests whose various solutions could specify the solver's approach or process. Through their emphasis on the process, research in problem solving was stimulated as well as the development of scoring procedures to aid in the characterization of process. Among their findings was the special ability of the successful problem solver to understand the nature of the problem and to attack it in its own terms. They found, on the other hand, that the unsuccessful problem solver lacked a sense of direction. Aptitude scores and grades in comprehensive examinations were their criteria for successful and unsuccessful problem solvers.

Earlier than Bloom and Broder, Karl Duncker (6) had studied the process of problem solving with his method of "thinking aloud" by the subjects. This method differed from introspection in that the subject's attention was directed to the problem rather than to his own thinking processes. This, too, supposedly was an improvement on retrospection since Duncker's technique did not depend on the subject's memory of his process. This, though, was also a qualitative approach.

Heidbreder (12) had studied adults and children in the problem solving situation very early, but the main interest here was to show that thought processes in general developed differentially through various ages. A limited conclusion was possible based on the reasons offered by the subjects for their responses.

Tate, Stanier, and Harootunian (26) constructed a battery of tests to distinquish good and poor problem solvers. One of their tests was the "Thought Problems," which is one of the criteria for this research. Their criteria received empirical validation by significant differences between good and poor problem solvers in "nearly all tests where quality of response, accuracy, or judgment is required." Another finding may be interesting in the light of the different contents of the two problems used in this research. They discovered that "the more complex the task or the more restricted the requirements, the greater their superiority (the good problem solvers)." Two basic problems with four variations each are used in the present study and in the light of the above remarks differential performance on the more difficult problem and variations should be observed.

B. The Instruments:

A technique similar to the one employed here was utilized by Bryan (2) and by Glaser, Damrin, and Gardner (9). Both were employed in the area of electronic trouble shooting.

Bryan's technique, called Automasts, offers choices of answers to the problems at different intervals. The result is that the obtained data does not give a true picture of the process as it takes place in the subject. An evaluation was made in terms of correct solutions, times, number of steps, use of clues, and guesses. This, therefore, is not really an evaluation of process.

Glaser, Damrin, and Gardner's method is referred to as the Tab Item Technique. This is applicable, also, beyond the area of electronic trouble shooting. The subject is presented with a malfunction, a series of possible check procedures, and the answers which are covered by tabs. Scoring methods for this device have not been clearly defined. One suggestion is the number of checks employed. Another was to weight the check procedures according to their relevance in isolating the defective unit. This last bears some resemblance to the techniques described in the next chapter, without, however, their more complete development.

The instrument used in this research was devised by Rimoldi (17) for the study of the diagnostic process in medical students. The focus was on the process of diagnosis rather than on the final diagnosis itself. This was accomplished, in general terms, by recording the information requested by the testee in his attempt to reach a solution. Several follow-up studies resulted: Rimoldi (18) on the process, Rimoldi, Devane, and Haley (19) on approaches to characterizing the process, and a summary of the whole medical study by Rimoldi, Haley, and Fogliatto (21).

The next step was the application of the technique in fields other than the medical. Examples of this are: Tabor (26); Mohrbacher (16); Gunn (11); Rimoldi, Meyer, Meyer, Fogliatto (23); Fogliatto (8); and Rimoldi, Haley, Fogliatto, Reyes, Erdmann, and Zacharia (20). These other areas include such fields as Rorschach interpretation, organic pathology in child guidance, and appraisal of personality parameters. A great deal of work has also been done with the instrument for the purpose of training in problem solving.

The exact type of instrument used in this study is described in Rimoldi, Haley, Fogliatto, Reyes, Erdmann, and Zacharia (20). "The subject is presented with a problem and a set of cards containing questions that he may ask in order to reach a solution. The subject is free to choose any card in any order he wishes. The corresponding answer to each question is written on the reverse side of the card. When the subject thinks that he has gained sufficient information to solve the problem correctly, he stops selecting cards and gives his solution. The experimenter records the questions in the order in which they were asked. These problems have no time limit."

The above study found this instrument to be valuable in distinquishing individual differences in problem solving ability- see also Fogliatto (7), in characterizing the process of the problem solver better than final solutions which cloud any individual differences, and in various aspects of using the problem as a training device. In short the instrument provided an experimental basis for the interpretation of the problem solving process. The instruments used in this research may be found in Appendix I of the above study (20).

C. Scoring Procedures:

One of the early methods of analyzing the performance (process) of a subject on the instrument used in this research may be called the group method. This procedure, as described in Rimoldi (17) utilizes the frequency of selection of a specific question in a specific order. These frequencies are then converted into proportions. Rimoldi, Haley, Fogliatto (21) note that these proportions may be proportions of the total number of responses or of the total number of possible responses i.e., the product of the number of questions and the number of subjects in the group. This latter consideration assumes that questions not asked also give valuable information. A table of proportions may be constructed and accordingly an individual sequence of questions evaluated to obtain a single score for each individual. The group method using the total number of possible responses is one of the procedures employed in this research. This method has been found valuable when one wishes to evaluate an individual in terms of a group, or to evaluate a performance in terms of a criterion group as in Rimoldi, Haley, and Fogliatto (21).

In a 1963 Loyola Psychometric Laboratory publication by Rimoldi, Haley, Fogliatto, and Erdmann (22) a new method of scoring was introduced. This method differed from the group method in the construction of the table of proportions. Once the table was established, the evaluation of a sequence was identical to the group method. This technique may be referred to as the schema method. This method established the norms (table of proportions) on the basis of the logical relationships or structure of the problem itself. The structure or schema of the problem dictates what possible sequences may occur. Given all the possible sequences, the tabulation of the frequencies for a particular question in a particular order is straightforward, and the conversion to proportions is accomplished as above. The rationale for this technique is based on the properties of the problem alone; therefore, an individual's score is based on an objective standard and not given in terms of how the rest of the group performed. The criterion for acceptability of a normative sequence is uniqueness and conformity to the schema or structure of the problem. Uniqueness refers to both questions and the order of the question. The schema method is the second scoring procedure under investigation.

The final scoring procedure has not as yet been set forth in publication but work has been done with it in the Loyola Psychometric Laboratory, and it appears to have great promise. This technique rests on the norms that are established by the schema or structure of the problem. Therefore, the table of proportions for this technique and for the schema method would be identical. The two methods differ in their application of the norms to the individual sequence. This will be discussed at length in the next chapter. This third procedure is called the pulling-out method.

CHAPTER IV.

DESCRIPTION AND APPLICATION OF THE THREE SCORING PROCEDURES

A. Group Method

The group method was chronologically the first scoring procedure used in the evaluation of performance on the instrument used in this research. Scores based on this method evaluate individual performance in terms of a group. This group may either be the group which had attempted a solution of the problem in question or it may be an outside group which is used as a criterion group, e.g., a panel of experts. Any estimation of individual performance, therefore, must be done in terms of some group and is therefore a relative estimation.

The first step in the application of the group method of scoring is the construction of the table of frequencies. Since every card or question in a problem is numerically identified and since the sequence of questions used by each subject has been recorded, it is possible to determine the number of times each question has been selected in a particular order by the entire group. When this frequency has been computed for every question in every order utilized (the orders cannot, of course, exceed the number of questions in a given problem) then a table such as the one in Table 1 will result.

	Questions				
ORDER	1	2	3	4	Σ
1	20	3	1	1	25
2	4	16	2	0	22
3	1	4	15	0	20
4	0	0	4	13	17
0	0	2	3	11	16
Σ	25	25	25	25	100

Table 1. Table of Frequencies

	Questions				
ORDER	1	2	3	4	Σ
1	.20	.03	.01	.01	.25
2	.04	.16	.02	.00	.22
3	.01	.04	.15	.00	.20
4	.00	.00	.04	.13	.17
0	.00	.02	.03	.11	.16
Σ	.25	.25	.25	.25	1.00

Table 2. Table of Proportions

Notice that question #1 was chosen in the first order 20 times and question #4 was neither chosen second nor third by anyone in the group. The total or sum for the first row or order indicates the number of individuals in the group, i.e., 25. This is true because for anyone to perform on the instrument he must at least choose one card or question. Observe also the row on the order dimension marked "0". The frequencies in this row indicate the number of individuals who did not select that particular question at all in their sequence. The zero order, as it is called, is included so that some recognition is made in this procedure for the questions which are not asked. This idea assumes that not only what is asked is important for the evaluation of performance, but also what is not asked. Therefore, the total responses in this table is 100 or the total number of possible responses. The number of affirmative responses was 84 and the zero order responses 16. The total number of possible responses is equal to the product of the number of subjects in the group and the number of questions given in the problem. This number is used as the denominator for determining the proportions given in the cells of Table 2, thus accounting for the questions not asked in the ultimate evaluation of performance.

Table 2 is the conversion of table 1 into proportions and thus becomes the norms upon which the scoring of an individual sequence depends. A single value, therefore, may be given to any sequence of questions which an individual may follow by finding the sum of the proportions corresponding to the ordered questions given by the particular sequence. For example, the sequence 1,2,3,4 would have the value .64, or .20 + .16 + .15 + .13 = .64. In the same way the sequence 3,4,1,2 would give a score of .02, or .01 + .00 + .01 + .00 = .02. In this manner each subject was scored for the various problem used in this study to obtain evaluation of performance according to the group method.

In summary, this technique implies that importance should be given to the questions selected, the order of selection, and to zero order responses. Finally, this evaluation is done in terms of a group.

B. The Schema Method:

This technique, as stated above, bases the construction of the norms or table of proportions on ideal solutions to the problem. These ideal solutions are dictated by the logical relationships within the problem which can be graphically presented by a schema, sometimes called a tree. For further explanation of this analysis see Loyola Psychometric Laboratory publication No. 28, "A Program for the Study of Thinking" by Rimoldi, Fogliatto, Haley and Erdmann (21). Having established the ideal sequences, one considers them just as observed sequences in the group method. A table of frequencies and a table of proportions including the zero order is computed using as a basis all the ideal sequences. The scoring of an individual observed sequence, then, is accomplished exactly as in the group method with the values coming from the norms or table of proportions generated by the ideal sequences.

In this method, then, the individual's performance is evaluated according to an objective standard which has been established by the logical relationships found in the specific problem. Consideration of the zero order is included also in this technique. Theoretically, its advantage over the group method lay in the fact that a score in this method need not be viewed from a relative standpoint.

C. The Pulling-Out Method:

This technique uses the same norms as the schema method and differs from it only in the application of the norms to the individual observed sequence. This method attempts to account for any restructuring or "late" understanding of the nature of the problem by the performer. In other words the benefit of the doubt is given to the subject in the evaluation of his performance.

The procedure involves a kind of matching of the observed sequence with one of the ideal sequences. That is, the scorer determines the ideal sequence which best approximates the observed sequence and will therefore maximize the evaluation of the performance. Obviously there are certain rules according to which this is done.

The first step is to remove all the irrelevant (as far as the ideal sequence is concerned) questions from the observed sequence. It is important to maintain the order of the questions as selected by the subject.

What results may be a complete or partial ideal sequence. In order to be complete,the order of the relevant observed questions must duplicate the ideal sequence. If this occurs, then one finds the value of the ideal sequence which would maximize the score for the observed sequence. This completes the second step in the determination of a final score for the pulling-out method. The third and final step is to divide the value, found at the completion of the second step, by the number of questions of the original observed sequence, i.e., before any pulling-out of irrelevant questions.

The sequence resulting from the pulling-out of irrelevant questions, however, may only partially duplicate an ideal sequence. In this case credit is given for the partial sequence. This value is again divided by the number of questions of the original observed sequence to determine the final score.

An example of the technique is in order to clarify the application. Suppose the observed sequence 1,6,3,8,2,10. Assume that the ideal sequences of the problem are 6,3,10 and 10,3,6. Pulling-out the irrelevant questions leaves 6,3,10 for the observed sequence. This exactly duplicates the ideal sequence 6,3,10 so the final score is the value of the 6,3,10 sequence in the schema norms divided by 6 (the number of questions from the original observed sequence). Had the original sequence been 1,10,8,3,2,6 then the ideal sequence 10,3,6 would have been duplicated with results exactly as above.

In most instances the ideal sequence will not be exactly duplicated. Assuming the observed sequence 1,6,7,8,2,3,5, the ideal sequence approximating it best is 6,3,10. However, there is only partial approximation here, namely 6,3. The final score is, therefore, the value of 6,3 in the schema norms, divided by 7 (in this case). The remnants of the observed sequence following the pulling-out of irrelevant questions must follow the order of one of the ideal sequences so that an observed sequence without 3 and 6 in it would obtain no value at all. If either occurred at the end of the sequence only that question would contribute any value. For instance the observed sequence 1, 3, 8, 4, would have zero as a final score. The sequence 1, 3, 6, 5, 7 would have the value of 6 in the first position in the schema norms divided by 5.

This technique, in summary, works to the advantage of the subject by giving him the benefit of the doubt as far as the occurrence of restructuring or reshaping the problem is concerned. It also incorporates the advantages of the schema method and adds the feature of differentially penalizing the subject for the prodigal selection of cards.

CHAPTER V.

PROCEDURE

A. Subjects:

The subjects used in this study were twenty-two freshmen (male) students of Loyola University, Chicago, Illinois. All the subjects participated in the research on a volunteer basis. As will be seen from the design of the research, it was not necessary to attempt to obtain any particular sample representative of a population.

B. Instruments:

a) General description.

The instruments* employed in this research are two types of problems, with four variations of each type. Therefore, a total of eight problems were administered. They were problems 31A, 31B, 31C, 31D, and 35A, 35B, 35C, 35D. All eight problems are of the type that presents a verbal definition of a problem situation together with a series of questions printed on separate cards. Each card contains a question on one side and the answer on the reverse side. The questions and answers contain information relevant to the problem situation, some of which is necessary for reaching a solution. The subject, in pursuing a solution, selects the cards he feels will give him the information needed for a solution and also records the order in which he chose to have the various questions answered. This establishes a sequence for each individual which describes his process and, also, supplies sufficient information for the experimenter to apply the three scoring procedures.

In identifying the problems, the number refers to a particular type of schema or framework or set of logical relationships upon which is superimposed various contents, identified by the letters. "A" presents the problem in concrete every-day situations. "B" presents the problem in abstract language or by means of letters that represent symbolically, non-specified concrete objects. In "C" the letters are presented in a negative manner. In all three of these forms of the one structure or schema, the answers are given in numbers. "D" is similar to form "B" in that letters stand for non-specified concrete objects in the questions, but, the answers are given in letters. The idea of quantity is, therefore, algebraic rather than numerical in this last form.

One will notice also that two different types of problem structure, namely 31 and 35, are used. The two structures represent a rather simple (31) and somewhat more complex type of problem (35).

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* These instruments are reproduced in full in Training in Problem Solving (Rimoldi, Fogliatto, Haley, Reyes, Erdmann, and Zacharia, 1962) Publication No. 27.

It is possible, on the basis of the foregoing, to define two levels of difficulty. One, called intrinsic, refers to the level of difficulty describing the complexity of the structure or logical relationships of the problem. The other refers to the content superimposed on the structure and describes its level of familiarity. This latter is called extrinsic difficulty.

Since it is not the express purpose of this research to study extrinsic and/or intrinsic difficulty, one may wonder why examples representing the various possibilities were included. The answer lies in the attempt to study the sensitivity of the three scoring procedures in a situation which would reflect the various instruments to which they might be applied. Therefore, the analysis of the findings will not be focused expressly on the two structures and their forms but on their interaction with the scoring procedures.

b) Administration.

Each of the eight problems were administered to the twenty-two subjects individually. They received the tests in numerical-alphabetical order, beginning with problem 31A. This problem represents the simplest structure and form (content) of the group. Each individual had as much time as he wished for each problem and only as many problems were administered at one session as could be comfortably handled by the subject in one hour. The testing continued at intervals of a week until all eight problems had been administered to each individual.

C. Methodology and Design:

The twenty-two subjects were divided into an upper and lower group with an equal number in each. The division was made using as criteria, the Raven's Progressive Matrices Tests and Thought Problems, Part 1. Some provision was attempted, therefore, for both reasoning and problem solving ability in the separation into two groups. Each subject was ranked according to a simple composite of the two criteria and the highest eleven formed the upper group with the others comprising the lower group. Since, according to the design, the two groups did not have to be significantly different - no check was necessary on the method of division. This formation of the groups was done subsequent to the collection and scoring of the data so as to introduce no bias in those processes.

The performance of each subject, as indicated by his sequence of questions, was scored in each of the three procedures under investigation. For each subject, then, there was a total of 24 scores, 8 for each procedure since he had taken 8 problems. Further, this means that each problem for any one subject had three scores, one reflecting the evaluation for each method. For each group there were 264 scores. This represented the performance of 11 subjects in 8 problems being scored three different ways.

At this point a four-way classification of the analysis of variance was computed for each group to determine possible significance for the main effects and pertinent interactions. The main effects, rows, columns, blocks, and squares, were subjects of a group, forms of a problem, the two structures, and the three scoring procedures respectively. The row dimension was an irrelevant dimension since it involved individual differences and these were not of interest as such in the research. The variance involved in this dimension, of course, proved quite valuable when considering the error estimate for measures of significance.

The finding of a significant variance for the main effect scoring procedures (squares) would further indicate that a "t" test for differences between the two groups on each scoring procedure would be meaningful. To distinquish the relative discriminative ability of each scoring procedure, it would not be necessary to find significant differences between the upper and lower groups but only to observe differences between these groups and assess the magnitude of these differences for each procedure.

Finally, the various interactions were graphed to describe the differential sensitivity of the scoring procedures with the simple and complex structures under the various forms A,B,C, and D.

CHAPTER VI

ANALYSIS AND DISCUSSION OF THE FINDINGS

A. Results of Analysis of Variance:

In general the results of the analysis of variance for both groups indicate that the variance attributable to the scoring procedure dimension (squares) is highly significant in each case. This indicates that for both the upper and lower group, evaluations of performance by the three scoring procedures would differ widely.

Specifically, the relevant information relating the results of the analysis of variance for the upper and lower group, is presented in tables 3 and 4, respectively. Table 3 indicates significant variance for the main effects, forms of the test (columns), and scoring methods (squares), both beyond the .001 level of significance. This indicates that for the upper group, performance is sensitive to the various forms of these tests, and suggests the possibility of successful differentiation of performance by these forms at both levels of intrinsic complexity as defined by the two schemata. As alluded to above, the high significance demonstrated by the scoring procedures dimension shows that a different description of performance may be expected by the application of the various methods to this data. The "F" ratio for the two types of schemata (blocks) did not approach any significant level, however, and thus suggests that for this superior group no differentiation by means of tests can be expected between these different levels of intrinsic difficulty. This is very interesting in the light of the findings for the inferior group where the "F" ratio was found to be significant, thus implying differentiation for the performance of this group between the two intrinsic levels of difficulty. Otherwise, the main effects of the lower group showed results similar to the upper group, except that in the lower group the levels of significance only reached the .01 level for the forms (A, B, C, D) dimension, suggesting that perhaps poorer performance on the various forms tends to be more homogeneous than better performance which may be more sensitive to differences in extrinsic difficulty.

TABLE 3

RESULTS OF ANALYSIS OF VARIANCE FOR UPPER GROUP

Source	Sum of Squares	df	Variance Estimate	F Ratio	Level of Significance
Main effects:					
R(Subjects)	.01445	10	.00144		
C(forms A,B,C,D)	.05752	3	.01917	17.4273	.001
B(Schemata)	.00007	1	.00007	.1014	none
S(Scoring Procedures)	.56737	2	.28368	578.9388	.001
Two-way Interaction:					
RC	.03311	30	.00110		
RB	.00693	10	.00069		
RS	.00972	20	.00049		
CB	.03688	3	.01229	7.2722	.001
CS	.08303	6	.01350	36.4865	.001
BS	.02872	2	.01436	57.44000	.001
Three-way Interaction:					
RCB	.05082	30	.00169		
RCS	.02224	60	.00037		
RBS	.00494	20	.00025		
CBS	.06082	6	.01014	23.58140	.001
Four-way Interaction:					
RCBS	.02587	60	.00043		

TABLE 4

RESULTS OF ANALYSIS OF VARIANCE FOR LOWER GROUP

Source	Sum of Squares	df	Variance Estimate	F Ratio	Level of Significance
Main Effects:					
R(Subjects)	.03700	10	.00370		
C(Forms A,B,C,D)	.01831	3	.00610	5.0833	.01
B(Schemata)	.01244	1	.01244	17.2778	.01
S(Scoring Procedures)	.64442	2	.32221	315.8922	.001
Two-way Interaction:					
RC	.03596	30	.00120		
RB	.00717	10	.00072		
RS	.02035	20	.00102		
CB	.02690	3	.00900	7.6271	.001
CS	.06990	6	.01165	29.1250	.001
BS	.06215	2	.03108	55.5	.001
Three-way Interaction:					
RCB	.03553	30	.00118		
RCS	.02421	60	.00040		
RBS	.01126	20	.00056		
CBS	.05066	6	.00844	22.2105	.001
Four-way Interaction:					
RCBS	.02312	60	.00038		

Returning to the analysis of the upper group we find significant variances for all the relevant interactions beyond $p < .001$. Remember that the subjects dimension (rows) involved individual differences and that, therefore, the relevant main effect (rows) and interactions were not of concern for this analysis. The forms by structures (columns x blocks) interaction indicates differences between the two sets of forms, one set for each schema or structure. The interaction is significant because the forms for one structure do not maintain a parallel relationship with the forms of the other structure. Provided all other things were equal except the intrinsic difficulty of the structure, one would expect this parallel relationship between the two structures with the various forms. However, training seems to play an important part here in that, as seen from the means of table 5 and corresponding graph in figure 1, the first acquaintance with the B form seems to have caused much more difficulty than the second meeting in 35B. As a matter of fact, performance on 35B and 35D were both better than performance on the previously experienced but simpler problem structure 31B and 31D. It thus appears that training overcame the increased level of intrinsic difficulty for the two schemata. This same phenomenon is similar in the lower group where significance is found for the forms by schemata (C x B) interaction also at the .001 level. But, as seen in table 6 and figure 2, the parallel relationship is almost maintained except for form B where the situation is like the upper group. Here, however, the difference in group ability seems to have precluded the better performance in the D form (35D) at the second acquaintance with this form. In general, considering both broups together, performance seems to be better on the less complex level of intrinsic difficulty, as would be expected from the nature of problem construction.

TABLE 5

MEANS OF THE INTERACTIONS OF FORMS (A,B,C,D) BY SCHEMATA (31,35) FOR THE UPPER GROUP

	Forms			
Schemata	A	B	C	D
M31	.10144	.04249	.10399	.08808
M35	.06999	.07708	.09668	.08810

TABLE 6

MEANS OF THE INTERACTIONS OF FORMS (A,B,C,D) BY SCHEMATA (31,35) FOR THE LOWER GROUP

	Forms			
Schemata	A	B	C	D
M31	.09799	.06154	.10666	.09482
M35	.06447	.08095	.08122	.07945

FIGURE 1

FORMS BY SCHEMATA INTERACTION FOR THE UPPER GROUP

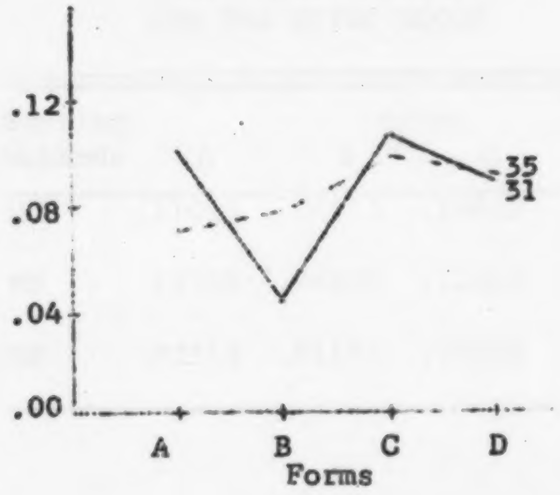
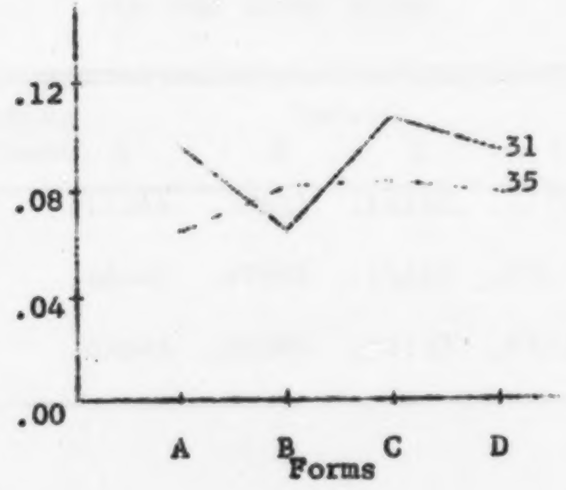


FIGURE 2

FORMS BY SCHEMATA INTERACTION FOR THE LOWER GROUP



The interaction forms of the structure by scoring procedures (C x S) also, had a significant "F" ratio for both groups beyond $p < .001$. Here the interaction as seen in table 7 and figure 3 for the upper group, and in table 8 and figure 4 for the lower group, seems to be quite similar. The obvious exception is the performance on form B of the lower group as described by the group method. This finding seems to lend credence to the interpretation of the lower level of significance for the forms (columns) dimension where it was suggested that poorer performance would be more homogeneous than better performance. Since the group technique involves scoring according to the group performance, individuals in a poorer, more homogeneous group would receive higher scores because of the homogeneity of the group. The homogeneity of the poorer group may also be expected to increase as the difficulty (extrinsic) increases. This, then, may be suggested as the explanation for the continuous ascending curve of the poorer group according to the group method.

In general, in the forms by schemata interaction, the schema method and the pulling-out method tend to react very similarly. The fact that the pulling-out method is represented by flatter curves for both groups is merely an artifact of the scale in that each unit of difference in a score is more meaningful because of the division by the number of questions in the sequence. The majority of the interaction is accounted for by the different interpretation of performance by the group method among the various forms. A plausible explanation was offered for this above.

TABLE 7

MEANS OF THE INTERACTION OF SCORING METHODS (GROUP, SCHEMA, PULLING-OUT) BY FORMS (A,B,C,D) FOR THE UPPER GROUP

Scoring Methods	Forms A	B	C	D
MG	.11093	.10113	.14646	.17759
MS	.11308	.06649	.12243	.07544
MP	.03313	.01174	.03210	.01123

TABLE 8

MEANS OF THE INTERACTION OF SCORING METHODS (GROUP, SCHEMA, PULLING-OUT) BY FORMS (A,B,C,D) FOR THE LOWER GROUP

Scoring Methods	Forms A	B	C	D
MG	.11264	.12434	.14141	.17748
MS	.10499	.07551	.11689	.07170
MP	.02606	.01389	.02353	.01223

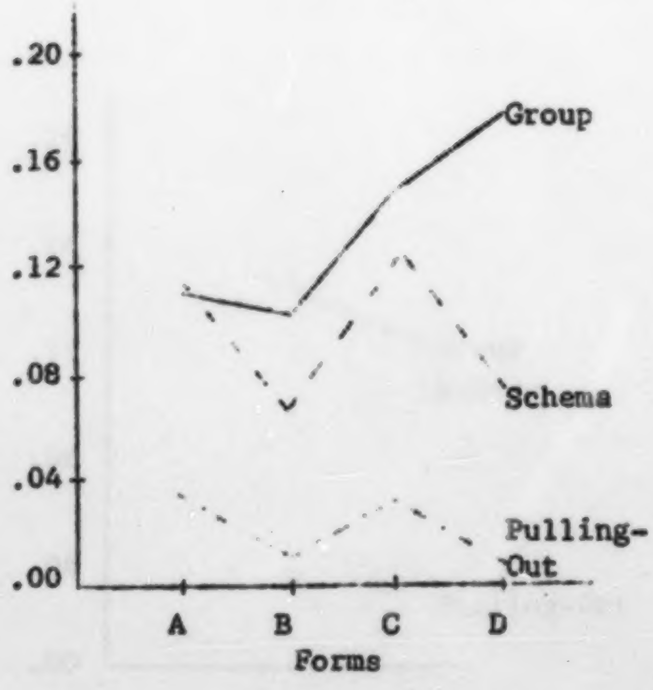


FIGURE 3

FORMS BY SCORING PROCEDURES INTERACTION FOR THE UPPER GROUP

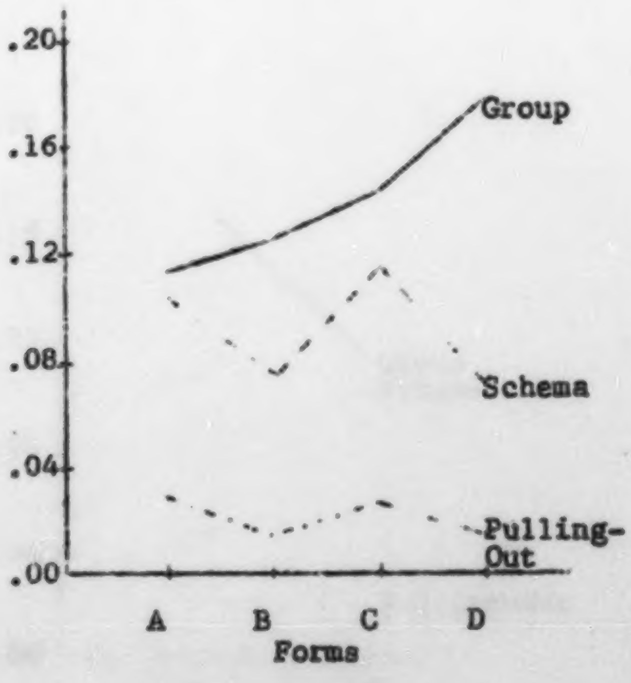


FIGURE 4

FORMS BY SCORING PROCEDURES INTERACTION FOR THE LOWER GROUP

TABLE 9

MEANS OF THE INTERACTION OF SCORING METHODS (GROUP, SCHEMA, PULLING-OUT) BY SCHEMATA (31, 35) FOR THE UPPER GROUP

Scoring Methods	Schemata 31	35
MG	.14701	.12105
MS	.08181	.10690
MP	.02317	.02093

TABLE 10

MEANS OF THE INTERACTION OF SCORING METHODS (GROUP, SCHEMA, PULLING-OUT) BY SCHEMATA (31,35) FOR THE LOWER GROUP

Scoring Methods	Schemata 31	35
MG	.16707	.11087
MS	.08465	.09989
MP	.01904	.01882

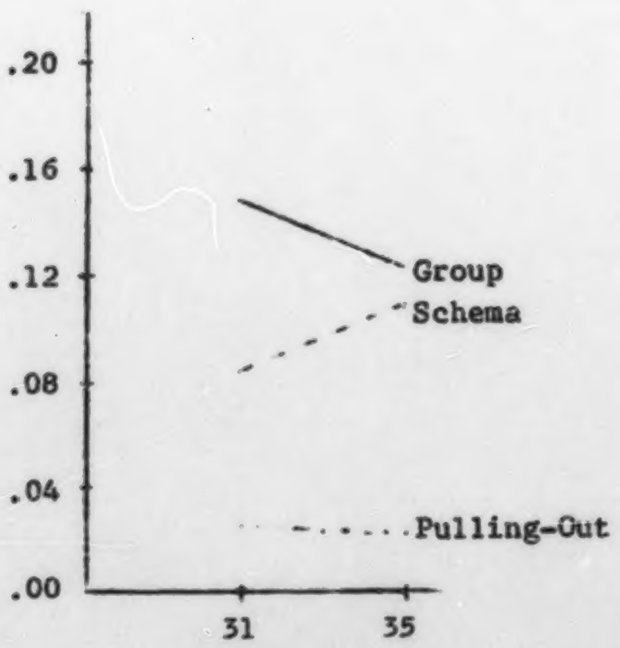


FIGURE 5

SCORING PROCEDURES BY SCHEMATA INTERACTION FOR THE UPPER GROUP

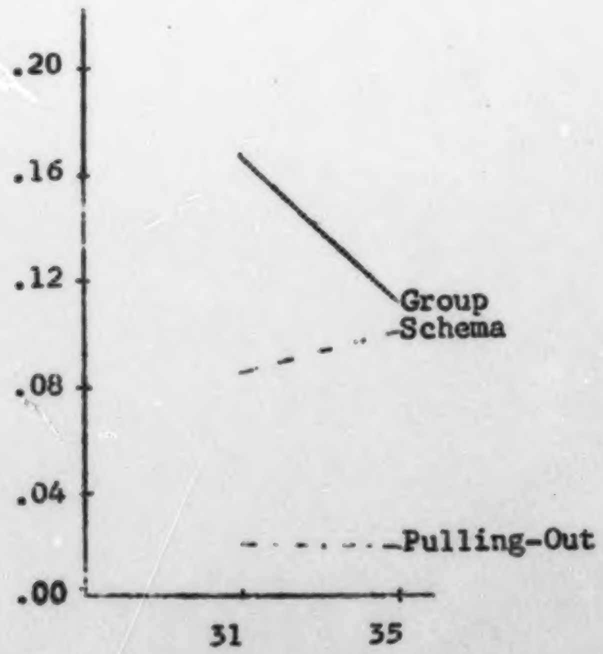


FIGURE 6

SCORING PROCEDURES BY SCHEMATA INTERACTION FOR THE LOWER GROUP

The third relevant interaction, structures by scoring procedures (B x S) is also significant beyond the .001 level for both groups. Table 9 and figure 5, and table 10 and figure 6 present the analysis of this interaction for the upper and lower groups respectively. The same type of relationship prevails for both groups. The group and pulling-out techniques both indicate lower performances for the more complex structure. The schema technique indicates the opposite result. In terms of what has been said formerly regarding the group technique, this may appear contradictory. However, it must be remembered that previously the analysis was for extrinsic difficulty whereas now the question revolves around intrinsic difficulty only. Here all differences due to forms have been suppressed because of the collapsing of the forms dimension. Before, in the forms by scoring methods (C x S) interaction, all differences due to structures had been suppressed because of the collapsing of the schemata dimension. Because of these considerations one would expect the phenomenon of homogeneity of poorer performance in the various forms to be suppressed. This occurs because the variance due to the forms of a schema does not contribute to this interaction (B x S). The picture presented by the schemata by scoring methods (B x S) interaction now limits the phenomenon of homogeneity of poor performance to a particular level of intrinsic difficulty. Perhaps familiarity with the structure may also account for some of the homogeneity.

There also remains the differences between the three techniques for the two structures. The dissenting description is offered by the schema method which imposes no penalty for the number irrelevant of questions asked. One would expect more irrelevant questions to be employed by the subjects in a structure of greater difficulty and if such is not considered by the technique this could easily account for the discrepancy found.

The remaining interaction, a three-way interaction, is also significant at the .001 level of confidence for both groups. As with almost all three-way interactions, interpretation is well nigh impossible and at the least dangerous and prone to error. Suffice it to say that a significant variance is found for the triple interaction of forms of the test, the two structures, and the three scoring techniques. Table 11 and figure 7, and table 12 and figure 8 present the data of this triple interaction.

B. Results of the "t" test:

To return to the express purpose of this study, a significant variance was attributable to the squares (scoring techniques) dimension. It was therefore meaningful, since they differed among themselves as evaluators of performance, to determine which is the most sensitive to differences between the two groups. For this purpose a one-tailed "t" test of significance was run between the two groups on each scoring procedure to determine and assess differences according to the particular procedure. The formula for "t" applied was that applied in the case of the difference of means for uncorrelated groups. The error estimate utilized was obtained from the variance estimates of the subjects by scoring methods interaction in the analysis of variance as suggested by McNemar (14).

The results are shown in Table 13 below where it can be seen that the pulling-out technique is the procedure which is most sensitive discriminator between the upper and lower groups, as established by the criteria of selection.

TABLE 13

MEAN DIFFERENCES, "t" VALUES, AND LEVELS OF SIGNIFICANCE FOR THE THREE SCORING METHODS

Methods	Mean Differences	"t" Values	Levels of Significance
Group	-.03950	-3.214	none
Schema	.01670	1.359	none
Pulling-Out	.02497	2.032	$P < .05$

The pulling-out technique shows a significant difference between the groups in the expected direction with the .05 level of confidence, and almost reaches the .025 level. The schema method shows a difference in the expected direction without, however, being significant. Its "t" value of 1.359 just misses the value 1.725 needed for the .05 level.

The group method does not even show a difference in the expected direction. As a matter of fact, it indicates that the lower group should really be called the superior group.

These results then, indicate that use of the group method should be confined to those situation when one is interested in evaluating the performance of individuals in one group in terms of the performance of another group or a criterion group.

Finally, the results of this research indicate that, under the limitations imposed by this sample, the pulling-out method of scoring discriminates best among the three procedures between good and poor problem solvers as defined by the specified criteria.

TABLE 11

MEANS OF THE INTERACTION OF THE SCORING METHODS (GROUP, SCHEMA, PULLING-OUT) BY FORMS (A,B,C,D) BY SCHEMATA (31,35) FOR THE UPPER GROUP

	Schemata							
	31				35			
	Forms				Forms			
Scoring Methods	A	B	C	D	A	B	C	D
MG	.13121	.08930	.14345	.22409	.09065	.11297	.14948	.13110
MS	.12727	.03409	.12727	.03863	.09889	.09889	.11760	.11225
MP	.04583	.00409	.04124	.00151	.02042	.01939	.02296	.02094

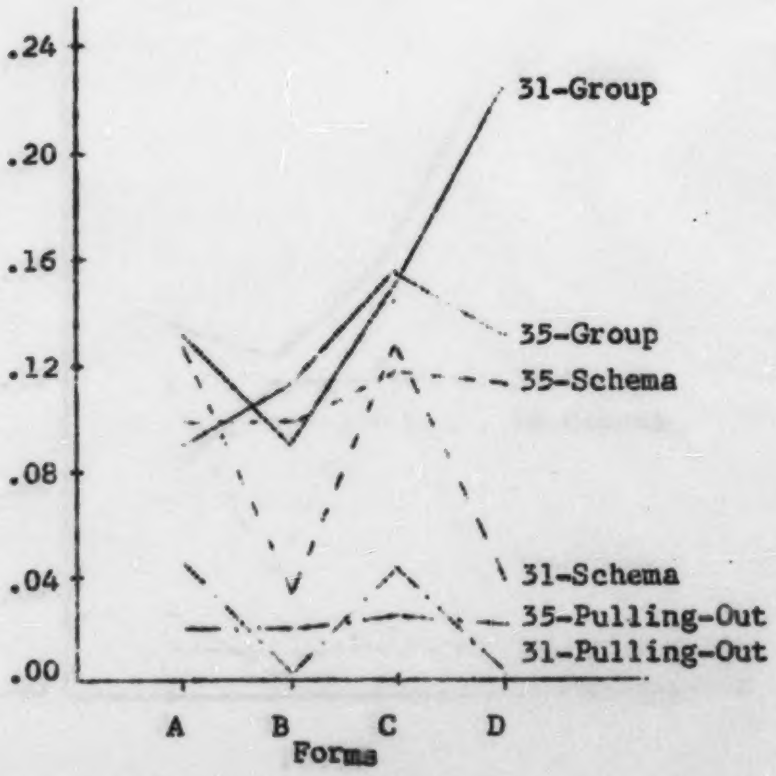


FIGURE 7

FORMS BY SCHEMATA BY SCORING PROCEDURES INTERACTION FOR THE UPPER GROUP

TABLE 12

MEANS OF THE INTERACTION OF THE SCORING METHODS (GROUP, SCHEMA, PULLING-OUT) BY FORMS (A,B,C,D) BY SCHEMATA (31,35) FOR THE LOWER GROUP

	Schemata							
	31				35			
	Forms				Forms			
Scoring Methods	A	B	C	D	A	B	C	D
MG	.14069	.12921	.16241	.23595	.08460	.11947	.12040	.11900
MS	.12045	.04545	.12954	.04318	.08953	.10557	.10423	.10022
MP	.03283	.00996	.02803	.00533	.01929	.01782	.01902	.01913

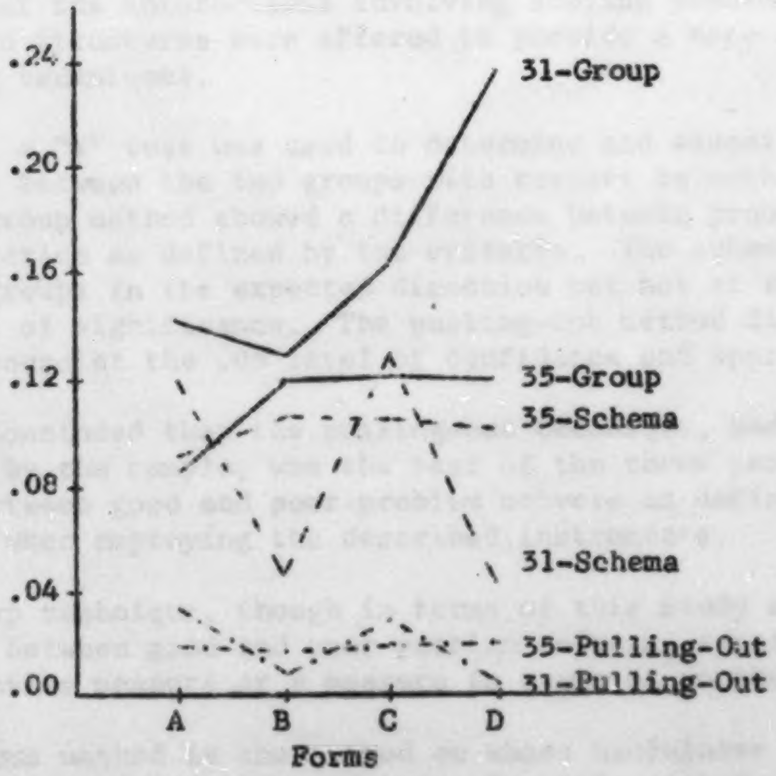


FIGURE 8

FORMS BY SCHEMATA BY SCORING PROCEDURES INTERACTION FOR THE LOWER GROUP

CHAPTER VII

SUMMARY

The purpose of the research was to investigate three scoring procedures as discriminators between good and poor problem solvers.

Twenty-two undergraduate males (freshmen) from Loyola University, Chicago, participated in the study. This sample was divided into two equal groups, an upper and a lower. The assignment to groups was done by means of two criteria, the Raven's Progressive Matrices Test and Thought Problems Part I, tapping reasoning and problem solving ability respectively. Two structures with four variations in content each, a total of eight problems, were administered to both groups in identical circumstances. The performance of each subject in each problem was then scored according to the three procedures.

An analysis of variance, four-way classification was then employed on the data for each group. Significant "F" ratios were found for both groups in the squares (scoring procedures) dimension beyond the .001 level of confidence. Significant variances were also found for the remaining relevant main effects and interactions of both groups with the exception of the block main effects (the two structures) for the upper group. Interpretations of the interactions involving scoring procedures, content variations, and structures were offered to provide a more complete analysis of the scoring techniques.

Finally, a "t" test was used to determine and assess the magnitude of differences between the two groups with respect to each scoring procedure. The group method showed a difference between groups but it was in the wrong direction as defined by the criteria. The schema method differentiated the groups in the expected direction but not at any commonly accepted level of significance. The pulling-out method did discriminate between the groups at the .05 level of confidence and approached the .025 level.

It was concluded that the pulling-out technique, under the limitations imposed by the sample, was the best of the three procedures in discriminating between good and poor problem solvers as defined by the specified criteria when employing the described instruments.

The group technique, though in terms of this study not valuable as a discriminator between good and poor problem solvers, retains its usefulness as a relative measure or a measure in terms of another criterion.

The schema method is the method on whose usefulness doubt is cast; but it will be remembered that the norms for this method are the same as for the pulling-out method, and that it is only in terms of the application of the norms to the sequence that they differ. Therefore, the usefulness of the application of the schema norms in the schema method is called into question by this study. The group method and the pulling-out method both have fruitful possibilities for future investigation.

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